# Chance and the predictive limit in basketball (both college and professional) 

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#### Abstract

There seems to be an upper limit to predicting the outcome of matches in (semi-)professional sports. A number of works have proposed that this is due to chance and attempts have been made to simulate the distribution of win percentages to identify the most likely proportion of matches decided by chance. We argue that the approach that has been chosen so far makes some simplifying assumptions that cause its result to be of limited practical value, especially for settings where teams do not play all possible opponents. Instead, we propose to use clustering of statistical team profiles and observed scheduling information to derive limits on the predictive accuracy for particular seasons, which can be used to assess the performance of predictive models on those seasons. Using NCAA basketball data, we show that the resulting simulated distributions are much closer to the observed distributions and give higher assessments of chance and tighter limits on predictive accuracy. We also show similar results for the NBA.


## 1 Introduction

In prior work on the topic of NCAA basketball [14], we speculated about the existence of a "glass ceiling" in (semi-)professional sports match outcome prediction, noting that season-long accuracies in the mid-seventies seemed to be the best that could be achieved for college basketball, with similar results for other sports. One possible explanation for this phenomenon is that we are lacking the attributes to properly describe sports teams, having difficulties to capture player experience or synergies, for instance. While this is the focus of on-going work in the community, especially for "under-described" sports such as European soccer or NFL football, we consider a different question in this paper: the influence of chance on match outcomes.

Even if we were able to accurately describe sports teams in terms of their performance statistics, the fact remains that athletes are humans, who might make mistakes and/or have a particularly good/bad day, that matches are refereed by humans, see before, that injuries might happen during the match, that the interaction of balls with obstacles off which they ricochet quickly becomes too complex to even model etc. Each of these can affect the match outcome
to varying degrees and especially if we have only static information from before the match available, it will be impossible to take them into account during prediction.

While this may be annoying from the perspective of a researcher in sports analytics, from the perspective of sports leagues and betting operators, this is a feature, not a bug. Matches of which the outcome is effectively known beforehand do not create a lot of excitement among fans, nor will they motivate bettors to take risks.

Intuitively, we would expect that chance has a stronger effect on the outcome of a match if the two opponents are roughly of the same quality, and if scoring is relatively rare: since a single goal can decide a soccer match, one (un)lucky bounce is all it needs for a weaker team to beat a stronger one. In a fast-paced basketball game, in which the total number of points can number in the two or even three hundreds, a single basket might be the deciding event between two evenly matched teams but probably not if the skill difference is large.

For match outcome predictions, a potential question is then: "How strong is the impact of chance for a particular league?", in particular since quantifying the impact of chance also allows to identify the "glass ceiling" for predictions. The topic has been explored for the NFL in [2-4], which reports

The actual observed distribution of win-loss records in the NFL is indistinguishable from a league in which $52.5 \%$ of the games are decided at random and not by the comparative strength of each opponent.

Using the same methodology, Weissbock et al. [13] derive that $76 \%$ of matches in the NHL are decided by chance. As we will argue in the following section, however, the approach used in those works is not applicable to NCAA basketball.

Before we continue, a short remark on terminology: Burke uses the term "luck" but we prefer the term "chance" since "luck" implies a positive outcome, whereas "chance" is meant to indicate randomness.

## 2 Identifying the impact of chance by Monte Carlo simulations

The general idea used by Burke and Weissbock ${ }^{1}$ is the following:

1. A chance value $c \in[0,1]$ is chosen.
2. Each out of a set of virtual teams is randomly assigned a strength rating.
3. For each match-up, a value $v \in[0,1]$ is randomly drawn from a uniform distribution.

- If $v \geq c$, the stronger team wins.
- Otherwise, the winner is decided by flipping an unweighted coin.

4. The simulation is re-iterated a large number of times (e.g. 10, 000) to smooth results.
[^0]Figure 1 shows the distribution of win percentages for 340 teams, 40 matches per team (roughly the settings of an NCAA basketball season including playoffs), and 10,000 iterations for $c=0.0$ (pure skill), $c=1.0$ (pure chance), and $c=0.5$.


Fig. 1. MC simulated win percentage distributions for different amounts of chance

By using a goodness of fit test $-\chi^{2}$ in the case of Burke's work, $F$-Test in the case of Weissbock's - the $c$-value is identified for which the simulated distribution fits the empirically observed one best, leading to the values reproduced in the introduction. The identified $c$-value can then be used to calculate the upper limit on predictive accuracy in the sport: since in $1-c$ cases the stronger team wins, and a predictor that predicts the stronger team to win can be expected to be correct in half the remaining cases in the long run, the upper limit lies at:

$$
(1-c)+c / 2
$$

leading in the case of

- the NFL to: $0.475+0.2625=0.7375$, and
- the NHL to: $0.24+0.36=0.62$

Any predictive accuracy that lies above those limits is due to the statistical quirks of the observed season: theoretically it is possible that chance always favors the stronger team, in which case predictive accuracy would actually be 1.0. As we will argue in the following section, however, NCAA seasons (and not only they) are likely to be quirky indeed.

## 3 Limitations of the MC simulation for NCAA basketball

A remarkable feature of Figure 1 is the symmetry and smoothness of the resulting curves. This is an artifact of the distribution assumed to model the theoretical
distribution of win percentages - the Binomial distribution - together with the large number of iterations. This can be best illustrated in the "pure skill" setting: even if the stronger team were always guaranteed to win a match, real-world sports schedules do not always guarantee that any team actually plays against a representative mix of teams both weaker and stronger than itself. A reasonably strong team could still lose every single match, and a weak one could win at a reasonable clip. One league where this is almost unavoidable is the NFL, which consists of 32 teams, each of which plays 16 regular season matches (plus at most 4 post-season matches), and ranking "easiest" and "hardest" schedules in the NFL is an every-season exercise. Burke himself worked with an empirical distribution that showed two peaks, one before 0.5 win percentage, one after. He argued that this is due to the small sample size (five seasons).


Fig. 2. Observed distribution of win percentages in the NCAA, 2008-2013

The situation is even more pronounced in NCAA basketball, where 340+ Division I teams play at most 40 matches each. Figure 2 shows the empirical distribution for win percentages in NCAA basketball for six season (2008-2013). ${ }^{2}$ While there is a pronounced peak for a win percentage of 0.5 for 2008 and 2012, the situation is different for 2009, 2010, 2011, and 2013. Even for the former two seasons, the rest of the distribution does not have the shape of a Binomial distribution. Instead it seems to be that of a mix of distributions - e.g. "pure skill" for match-ups with large strength disparities overlaid over "pure chance" for approximately evenly matched teams.

NCAA scheduling is subject to conference memberships and teams will try to pad out their schedules with relatively easy wins, violating the implicit assumptions made for the sake of MC simulations. This also means that the "statistical

[^1]quirks" mentioned above are often the norm for any given season, not the exception. Thought to its logical conclusion, the results that can be derived from the Monte Carlo simulation described above are purely theoretical: if one could observe an effectively unlimited number of seasons, during which schedules are not systematically imbalanced, the overall attainable predictive accuracy were bound by the limit than can be derived by the simulation. For a given season, however, and the question how well a learned model performed w.r.t. the specificities of that season, this limit might be too high (or too low).


Fig. 3. Distribution of win percentages 2008

As an illustration, consider Figure 3, showing several season simulations as well as observed winning percentages for the 2008 season. ${ }^{3}$ Differing from Burke and Weissbock, we use neither the $\chi^{2}$ test (which requires frequency binning), to compare distributions, nor the $F$-Test, which only compares variances. Instead, we chose the Kolmogorov-Smirnov Test, which returns the maximum distances between two distributions' cumulative distribution functions (CDF).

The MC simulation that matches the observed proportion of teams having a win percentage of 0.5 is derived by setting $c=0.42$, implying that a predictive accuracy of 0.79 should be possible. The MC simulation that fits the observed distribution best, according to the Kolmogorov-Smirnov (KS) test (which overestimates the proportion of teams having a win percentage of 0.5 along the way), is derived from $c=0.525$ (same as Burke's NFL analysis), setting the predictive limit to 0.7375 . Both curves have visually nothing in common with the observed distribution, yet the null hypothesis - that both samples derive from the same distribution - is not rejected at the 0.001 level by the KS test for sample comparison. This hints at the weakness of using such tests to establish similarity:

[^2]CDFs and standard deviations might simply not provide enough information to decide whether a distribution is appropriate.

## 4 Related work

As mentioned above, Weissbock $[12,13]$ and Burke [2-4] use a Monte Carlo simulation to identify winning percentage distributions most similar to an observed one using a comparison measure and then derive the chance value. We have given the details of their approach in Section 2.

Academic work on the subject is relatively rare. Mauboussin [8] compares the variance of winning percentages of observed seasons to the variance of an all-chance season to derive the impact of chance. As a result of this analysis, he claims that the influence of chance on the NBA is only $12 \%$ because the skill difference between teams were so large. This is an astounding number since it would imply that one can achieve a predictive accuracy of $\mathbf{9 4 \%}$ ! Incidentally, this would make lucrative betting on the NBA impossible.

Aoki et al. [1] consider the distribution of points won at home and away for teams (for soccer, for instance, 3 for a win, 1 for a tie, 0 for a loss), which in an equal-skill (pure-chance) league would be normally distributed. By comparing the observed distribution's variance to the expected one, they derive a coefficient placing different leagues on a skill-chance spectrum. They report basketball as being least influenced by chance, followed by volleyball, soccer, and handball. They do not report concrete chance values, except for the NBA where they estimate it as being $35 \%$, but explore how many teams that are much better (or much worse) than the rest have to be removed to end up with a pure-chance league. Basketball (50\%) and volleyball ( $40 \%$ ) require many such removals, whereas in soccer $(19 \%)$ and handball ( $14 \%$ ) there are only few outlier teams.

Gilbert and Wells [7] place "luck" in the context of ludology, the study of complex games. Defining two measures, , they consider individual MLB (baseball), NFL, NHL, and NBA matches. They find NBA matches to be less affected by chance than NHL and MLB ones (due to larger skill differences in the NBA). They also report NFL matches to be least affected by discuss that the shorter season seems to cancel this out.

Sarkar and Kamath [11] consider soccer, and aim to establish whether there is a difference in "X-factors" between the first and last six teams in a season's ranking, and to what degree chance affects the final rankings. They compare expected (predicted) points, goals etc. to observed ones to derive what they term "X-factors". By comparing the mean of X-factors between consecutive positions in the ranking, they find that the X-factor has explanatory power for the first two positions, and that chance has no effect on the top-six.

Csurilla et al. [5] consider the results of $3 \times 3$ and $5 \times 5$ basketball world cups, using the final ranking as ground truth. Using four chance measures, they find that women's basketball competitions are less influenced by chance than men's, and that $3 \times 3$ is more influenced by chance than $5 \times 5$. They do not report concrete chance values.

## 5 Deriving limits for specific seasons

The ideal case derived from the MC simulation does not help us very much in assessing how close a predictive model comes to the best possible prediction. Instead of trying to answer the theoretical question: What is the expected limit to predictive accuracy for a given league?,
we therefore want to answer the practical question: Given a specific season, what was the highest possible predictive accuracy?.

To this end, we still need to find a way of estimating the impact of chance on match outcomes, while taking the specificities of scheduling into account. The problem with estimating the impact of chance stays the same, however: for any given match, we need to know the relative strength of the two teams but if we knew that, we would have no need to learn a predictive model in the first place. If one team has a lower adjusted offensive efficiency than the other (i.e. scoring less), for example, but also a lower adjusted defensive efficiency (i.e. giving up fewer points), should it be considered weaker, stronger, or of the same strength?

Learning a model for relative strength and using it to assess chance would therefore feed the models potential errors back into that estimate. What we can attempt to identify, however, is which teams are similar.

### 5.1 Clustering team profiles and deriving match-up settings

| Offensive stats |  | Defensive stats |  |
| :---: | :---: | :---: | :---: |
| AdjOEff | Points per 100 possessions scored, <br> adjusted for opponent's strength | AdjDEff | Points per 100 possessions allowed, <br> adjusted for opponent's strength |
| OeFG\% | Effective field goal percentage | DeFG\% | eFG\% allowed |
| OTOR | Turnover rate | DTOR | TOR forced |
| OORR | Offensive rebound rate | DORR | ORR allowed |
| OFTR | Free throw rate | DFTR | FTR allowed |

Table 1. Statistics used to described teams for clustering, definitions can be found at www.basketball-reference.com/glossary

We describe each team in terms of their adjusted efficiencies, and their Four Factors, adopting Ken Pomeroy's representation [10]. Each statistic is present both in its offensive form - how well the team performed, and in its defensive form - how well it allowed its opponents to perform (Table 1). We use the averaged end-of-season statistics, leaving us with approximately 340 data points per season. Clustering daily team profiles, to identify finer-grained relationships, and teams' development over the course of the season, is left as future work. As a clustering algorithm, we used the WEKA [6] implementation of the EM algorithm with default parameters. This involves EM selecting the appropriate number of clusters by internal cross validation, with the second row of Table 2 showing how many clusters have been found per season.

As can be seen, depending on the season, the EM algorithm does not separate the 340 teams into many different statistical profiles. Additionally, as the third

| Season | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of Clusters (EM) | 5 | 4 | 6 | 7 | 4 | 3 |
| Cluster IDs in Tournament | 1,5 | 4 | 2,6 | $1,2,5$ | 3,4 | 2 |
| \# Clusters (Optimized EM, Section 7) | 20 | 4 | 19 | 20 | 14 | 13 |

Table 2. Number of clusters per season and clusters represented in the NCAA tournament (second line)
row shows, only certain clusters, representing relatively strong teams, make it into the NCAA tournament, with the chance to eventually play for the national championship (and one cluster dominates, like Cluster 5 in 2008). These are strong indications that the clustering algorithm does indeed discover similarities among teams that allow us to abstract "relative strength". Using the clustering results, we can re-encode a season's matches in terms of the clusters to which the playing teams belong, capturing the specificities of the season's schedule.

|  | Cluster 1 | Cluster 2 | Cluster 3 | Cluster 4 | Cluster 5 | Weaker opponent |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Cluster 1 | $76 / 114$ | $161 / 203$ | $52 / 53$ | $168 / 176$ | $65 / 141$ | $381 / 687(0.5545)$ |
| Cluster 2 | $100 / 176$ | $298 / 458$ | $176 / 205$ | $429 / 491$ | $91 / 216$ | $705 / 1546(0.4560)$ |
| Cluster 3 | $7 / 32$ | $55 / 170$ | $47 / 77$ | $119 / 194$ | $4 / 40$ | $119 / 513(0.2320)$ |
| Cluster 4 | $22 / 79$ | $161 / 379$ | $117 / 185$ | $463 / 769$ | $28 / 145$ | $117 / 1557(0.0751)$ |
| Cluster 5 | $117 / 154$ | $232 / 280$ | $78 / 83$ | $232 / 247$ | $121 / 198$ | $659 / 962(0.6850)$ |

Table 3. Wins / total matches for pairings of teams of the clusters indicated in the row and column of each cell, 2008

Table 3 summarizes the re-encoded schedule for $2008 .{ }^{4}$ The re-encoding allows us to flesh out the intuition mentioned in the introduction some more: teams from the same cluster can be expected to have approximately the same strength, increasing the impact of chance on the outcome. Since we want to take all non-chance effects into account, we encode pairings in terms of which team has home court. The left margin indicates which team has home court in the pairing: this means, for instance, that while teams from Cluster 1 beat teams from Cluster 2 almost $80 \%$ of the time when they have home court advantage, teams from Cluster 2 prevail in almost $57 \%$ of the time when home court advantage is theirs. The effect of home court advantage is particularly pronounced on the diagonal, where unconditional winning percentages by definition should be at approximately $50 \%$. Instead, home court advantage pushes them always above $60 \%$. One can also see that many teams - all those in clusters 3 and 4 mainly play against stronger teams, and the teams in cluster 5 mainly against weaker ones. In those cases, chance would need to intervene rather strongly to alter match outcomes.

[^3]Table 3 is the empirical instantiation of our remark in Section 3: instead of a single distribution, 2008 seems to have been a weighted mixture of 25 distributions. ${ }^{5}$ None of these specificities can be captured by the unbiased MC simulation.

### 5.2 Estimating chance

The re-encoded schedule includes all the information we need to assess the effects of chance. The win percentage for a particular cluster pairing indicates which of the two clusters should be considered the stronger one in those circumstances, and from those matches that are lost by the stronger team, we can calculate the chance involved.

Consider, for instance, the pairing Cluster 5 - Cluster 2. When playing at home, teams from Cluster 5 win this match-up in $82.85 \%$ of the cases! This is the practical limit to predictive accuracy in this setting for a model that always predicts the stronger team to win, and in the same way we used $c$ to calculate that limit above, we can now invert the process: $c=2 *(1-0.8285)=0.343$. When teams from Cluster 5 welcomed teams from Cluster 2 on their home court in 2008, the overall outcome is indistinguishable from $34.3 \%$ of matches having been decided by chance.

The impact of chance for each cluster pairing, and the number of matches that have been played in particular settings, finally, allows us to calculate the effect of chance on the entire season, and using this result, the upper limit for predictive accuracy that could have been reached for a particular season.

| Season | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Unconstrained EM |  |  |  |  |  |  |
| KS | 0.0526 | 0.0307 | 0.0506 | 0.0327 | 0.0539 | 0.0429 |
| Chance | 0.5736 | 0.5341 | 0.5066 | 0.5343 | 0.5486 | 0.5322 |
| Limit for predictive accuracy | 0.7132 | 0.7329 | 0.7467 | 0.7329 | 0.7257 | 0.7339 |
| Optimized EM (Section 7) |  |  |  |  |  |  |
| KS | 0.0236 | 0.0307 | 0.0396 | 0.0327 | 0.0315 | 0.0410 |
| Chance | 0.4779 | 0.5341 | 0.4704 | 0.5343 | 0.4853 | 0.5311 |
| Limit | 0.7610 | 0.7329 | 0.7648 | 0.7329 | 0.7573 | 0.7345 |
| KenPom prediction accuracy | 0.7105 | 0.7112 | 0.7244 | 0.7148 | 0.7307 | 0.7035 |
| ANN prediction accuracy from [14] | 0.7136 | 0.7357 | 0.7115 | 0.7248 | 0.7120 | 0.7187 |

Table 4. Effects of chance on different seasons' matches and limit on predictive accuracy (for team encoding shown in Table 1)

The upper part of Table 4 shows the resulting effects of chance and the limits regarding predictive accuracy for the six seasons under consideration. Notably, the last two rows show the predictive accuracy when using the method described

[^4]on [10] and results we have reported in [14], using an artificial neural network model. Ken Pomeroy's method uses the Log5-method, with Pythagorean expectation to derive each team's win probability, and the adjusted efficiencies of the home (away) team improved (degraded) by $1.4 \%$. This method effectively always predicts the stronger team to win and should therefore show similar behavior as the observed outcomes. Its accuracy is always close to the limit and in one case (2012) actually exceeds it. One could explain this by the use of daily instead of end-of-season statistics but there is also another aspect in play. To describe that aspect, we need to discuss simulating seasons.

## 6 Simulating seasons

With the scheduling information and the impact of chance for different pairings, we can simulate seasons in a similar manner to the Monte Carlo simulations we have discussed above, but with results that are much closer to the distribution of observed seasons:

1. A chance value $c \in[0,1]$ is chosen.
2. A set of virtual teams is assigned cluster labels as derived by the EM clustering in such a manner that the distribution of instances to clusters matches the observed distribution.
3. For each match-up, a value $v \in[0,1]$ is randomly drawn from a uniform distribution.

- If $v \geq c$, the stronger team wins.
- Otherwise, the winner is decided by flipping a weighted coin, with the weight derived from the observed win probability for the cluster pairing.

4. The simulation is re-iterated a large number of times (e.g. 10, 000) to smooth results.

Figure 3 shows that while the simulated distribution is not equivalent to the observed one, it shows very similar trends. In addition, while the KS test does not reject any of the three simulated distributions, the distance of the one resulting from our approach to the observed one is lower than for the two Monte Carlo simulated ones.

The figure shows the result of simulating the season 10, 000 times, leading to the stabilization of the distribution. For fewer iterations, e.g. 100 or fewer, distributions that diverge more from the observed season can be created. In particular, this allows the exploration of counterfactuals: if certain outcomes were due to chance, how would the model change if they came out differently? Finally, the information encoded in the different clusters - means of statistics and co-variance matrices - allows the generation of synthetic team instances that fit the cluster (similar to value imputation), which in combination with scheduling information could be used to generate wholly synthetic seasons to augment the training data used for learning predictive models. We plan to explore this direction in future work.

## $7 \quad$ Finding a good clustering

Coming back to predictive limits, there is no guarantee that the number of clusters found by the unconstrained EM will actually result in a distribution of win percentages that is necessarily close to the observed one. Instead, we can use the approach outlined in the preceding section to find a good clustering to base our chance and predictive accuracy limits on:

1. We let EM cluster teams for a fixed number of clusters (we evaluated 4-20)
2. For a derived clustering, we simulate 10,000 seasons
3. The resulting distribution is compared to the observed one using the KolmogorovSmirnov score

| Season | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 0.0838 | $\mathbf{0 . 0 3 0 7}$ | 0.0560 | 0.0702 | 0.0539 | 0.0644 |
| 5 | 0.0710 | 0.0356 | 0.0543 | 0.0624 | 0.0609 | 0.0468 |
| 6 | 0.0488 | 0.0377 | 0.0506 | 0.0528 | 0.0404 | 0.0448 |
| 7 | 0.0494 | 0.0382 | 0.0433 | 0.0327 | 0.0390 | 0.0432 |
| 8 | 0.0466 | 0.0381 | 0.0417 | 0.0355 | 0.0404 | 0.0435 |
| 9 | 0.0534 | 0.0338 | 0.0424 | 0.0396 | 0.0398 | 0.0414 |
| 10 | 0.0564 | 0.0332 | 0.0421 | 0.0349 | 0.0370 | 0.0503 |
| 11 | 0.0478 | 0.0342 | 0.0447 | 0.0336 | 0.0325 | 0.0433 |
| 12 | 0.0326 | 0.0436 | 0.0545 | 0.0390 | 0.0378 | 0.0410 |
| 13 | 0.0357 | 0.0432 | 0.0559 | 0.0367 | 0.0351 | $\mathbf{0 . 0 4 0 2}$ |
| 14 | 0.0374 | 0.0384 | 0.0511 | 0.0342 | $\mathbf{0 . 0 3 1 5}$ | 0.0439 |
| 15 | 0.0385 | 0.0449 | 0.0578 | 0.0380 | 0.0350 | 0.0412 |
| 16 | 0.0388 | 0.0456 | 0.0570 | 0.0364 | 0.0361 | 0.0527 |
| 17 | 0.0269 | 0.0437 | 0.0480 | 0.0433 | 0.0441 | 0.0464 |
| 18 | 0.0293 | 0.0413 | 0.0441 | 0.0409 | 0.0327 | 0.0449 |
| 19 | 0.0276 | 0.0420 | $\mathbf{0 . 0 3 9 6}$ | 0.0392 | 0.0462 | 0.0462 |
| 20 | $\mathbf{0 . 0 2 3 6}$ | 0.0387 | 0.0460 | $\mathbf{0 . 0 2 8 9}$ | 0.0371 | 0.0493 |

Table 5. KS similarity between observed and simulated distributions for different numbers of clusters, lower is better, best values are indicated in bold

The results of this optimization are shown in Table 5. What is interesting to see is that a) increasing the number of clusters does not automatically lead to a better fit with the observed distribution, and b) clusterings with different numbers of clusters occasionally lead to the same KS, validating our comment in Footnote 5.

Based on the clustering with the lowest KS, we calculate chance and predictive limit and show them in the second set of rows of Table 4. EM already found the opimal assigment of teams to clusters for 2009 but for other seasons, there are quite a few more clusters. Generally speaking, optimizing the fit allows to lower the KS quite a bit and leads to lower estimated chance and higher
predictive limits. For both categories, however, the fact remains that different seasons were influenced by chance to differing degrees and therefore different limits exist. Furthermore, the limits we have found stay significantly below $80 \%$ and are different from the limits than can be derived from MC simulation.

Those results obviously come with some caveats:

1. Teams were described in terms of adjusted efficiencies and Four Factors adding or removing statistics could lead to different numbers of clusters and different cluster memberships.
2. Predictive models that use additional information, e.g. experience of players, or networks models for drawing comparisons between teams that did not play each other, can exceed the limits reported in Table 4.

The table also indicates that it might be less than ideal to learn from preceding seasons to predict the current one (the approach we have chosen in our previous work): having a larger element of chance (e.g. 2009) could bias the learner against relatively stronger teams and lead it to underestimate a team's chances in a more regular season (e.g. 2010).

## 8 NBA results

Finally, we apply the same approach to NBA seasons. The NBA contains much fewer teams (30), and plays many more matches (82), with the result that every team plays every other one at least twice.

| Season | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Optimized EM (Section 7) |  |  |  |  |  |  |  |  |
| Number of clusters | 5 | 6 | 5 | 9 | 6 | 10 | 9 | 7 |
| KS | 0.0749 | 0.0693 | 0.0848 | 0.0802 | 0.0618 | 0.0822 | 0.0512 | 0.0840 |
| Chance | 0.5821 | 0.5779 | 0.5930 | 0.5802 | 0.6091 | 0.5841 | 0.6005 | 0.6066 |
| Limit | 0.7090 | 0.7110 | 0.7035 | 0.7099 | 0.6954 | 0.7080 | 0.6998 | 0.6967 |
| KenPom prediction | 0.6725 | 0.6920 | 0.6700 | 0.6697 | 0.6601 | 0.6530 | 0.6475 | 0.6590 |
| NB prediction accuracy from [14] | 0.6608 | 0.6494 | 0.6506 | 0.6331 | 0.6187 | 0.6240 | 0.6438 | 0.6545 |

Table 6. Effects of chance on different NBA seasons' matches and limits on predictive accuracy

The results can be seen in Table 6 . Compared to Table 4, chance values are higher, and predictive limits lower. This is expected since talent differences between NBA teams are less than for NCAAB teams (only the best NCAAB players end up in the NBA after all), which allows chance to have a bigger effect, as we discussed in the introduction. We also see that the prediction accuracies of the predictive models are lower, most notably for Ken Pomeroy's model, which, as mentioned before, effectively always predicts the best team to win.

## 9 Summary and conclusions

In this paper, we have considered the question of the impact of chance on the outcome of (semi-)professional sports matches in more detail. In particular, we have shown that the unbiased MC simulations used to assess chance in the NFL and NHL are not applicable to the college basketball setting. We have argued that the resulting limits on predictive accuracy rest on simplifying and idealized assumptions and therefore do not help in assessing the performance of a predictive model on a particular season.

As an alternative, we propose clustering teams' statistical profiles and reencoding a season's schedule in terms of which clusters play against each other. Using this approach, we have shown that college basketball seasons violate the assumptions of the unbiased MC simulation, given higher estimates for chance, as well as tighter limits for predictive accuracy.

There are several directions that we intend to pursue in the future. First, as we have argued above, NCAA basketball is not the only setting in which imbalanced schedules occur. We would expect similar effects in the NFL, and soccer, which, as mentioned, is lower-scoring and where teams typically only play each other twice. What is needed to explore this question is a good statistical representation of teams, something that is easier to achieve for basketball than football/soccer teams.

In addition, as we have mentioned in Section 6, the exploration of counterfactuals and generation of synthetic data should help in analyzing sports better. We find a recent paper [9] particularly inspirational, in that the authors used a detailed simulation of substitution and activity patterns to explore alternative outcomes for an NBA playoff series.

Finally, since we can identify different cluster pairings and the differing of chance therein, separating those cases and training classifiers idependently for each could improve classification accuracy. To achieve this, however, we will need solve the problem of clustering statistical profiles over the entire season - which should also allow to identify certain trends over the course of seasons.

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## A Clustered schedules for different seasons, unconstrained EM

|  | Cluster 1 | Cluster 2 | Cluster 3 | Cluster 4 | Weaker opponent |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Cluster 1 | $133 / 197$ | $46 / 182$ | $105 / 272$ | $1 / 45$ | $0 / 696(0.0000)$ |
| Cluster 2 | $210 / 227$ | $231 / 352$ | $262 / 374$ | $76 / 247$ | $472 / 1200(0.3933)$ |
| Cluster 3 | $261 / 308$ | $192 / 357$ | $409 / 663$ | $56 / 261$ | $453 / 1589(0.2851)$ |
| Cluster 4 | $210 / 211$ | $341 / 374$ | $424 / 448$ | $515 / 818$ | $975 / 1851(0.5267)$ |

Table 7. Wins and total matches for different cluster pairings, 2009

|  | Cluster 1 | Cluster 2 | Cluster 3 | Cluster 4 | Cluster 5 | Cluster 6 | Weaker opponent |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cluster 1 | $129 / 204$ | $18 / 104$ | $6 / 14$ | $47 / 126$ | $33 / 145$ | $0 / 18$ | $0 / 611(0.0000)$ |
| Cluster 2 | $163 / 167$ | $269 / 437$ | $76 / 105$ | $255 / 292$ | $134 / 195$ | $73 / 249$ | $628 / 1445(0.4346)$ |
| Cluster 3 | $29 / 34$ | $64 / 95$ | $12 / 18$ | $49 / 58$ | $21 / 41$ | $30 / 87$ | $163 / 333(0.4895)$ |
| Cluster 4 | $109 / 136$ | $71 / 240$ | $19 / 46$ | $159 / 232$ | $55 / 119$ | $6 / 87$ | $109 / 860(0.1267)$ |
| Cluster 5 | $147 / 163$ | $87 / 166$ | $14 / 23$ | $101 / 123$ | $71 / 118$ | $17 / 57$ | $349 / 650(0.5369)$ |
| Cluster 6 | $120 / 120$ | $336 / 361$ | $100 / 117$ | $169 / 172$ | $133 / 141$ | $360 / 579$ | $858 / 1490(0.5758)$ |

Table 8. Wins and total matches for different cluster pairings, 2010

|  | Cluster 1 | Cluster 2 | Cluster 3 | Cluster 4 | Cluster 5 | Cluster 6 | Cluster 7 | Weaker opponent |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cluster 1 | $89 / 138$ | $140 / 174$ | $40 / 40$ | $69 / 73$ | $93 / 185$ | $97 / 103$ | $86 / 99$ | $525 / 812(0.6466)$ |
| Cluster 2 | $66 / 148$ | $235 / 369$ | $70 / 71$ | $141 / 167$ | $29 / 121$ | $166 / 206$ | $118 / 176$ | $495 / 1258(0.3935)$ |
| Cluster 3 | $2 / 14$ | $15 / 55$ | $29 / 39$ | $16 / 42$ | $0 / 8$ | $20 / 85$ | $4 / 31$ | $0 / 274(0.0000)$ |
| Cluster 4 | $10 / 48$ | $48 / 151$ | $36 / 40$ | $42 / 85$ | $2 / 28$ | $55 / 100$ | $28 / 68$ | $91 / 520(0.1750)$ |
| Cluster 5 | $166 / 217$ | $187 / 206$ | $43 / 43$ | $79 / 80$ | $205 / 339$ | $80 / 83$ | $148 / 160$ | $703 / 1128(0.6232)$ |
| Cluster 6 | $11 / 49$ | $76 / 178$ | $77 / 88$ | $72 / 97$ | $7 / 47$ | $94 / 151$ | $34 / 65$ | $183 / 675(0.2711)$ |
| Cluster 7 | $29 / 82$ | $97 / 160$ | $57 / 58$ | $59 / 72$ | $30 / 125$ | $74 / 92$ | $79 / 127$ | $287 / 716(0.4008)$ |

Table 9. Wins and total matches for different cluster pairings, 2011

|  | Cluster 1 | Cluster 2 | Cluster 3 | Cluster 4 | Weaker opponent |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Cluster 1 | $108 / 201$ | $110 / 320$ | $20 / 119$ | $19 / 121$ | $0 / 761(0.0000)$ |
| Cluster 2 | $362 / 416$ | $610 / 960$ | $105 / 354$ | $175 / 394$ | $362 / 2124(0.1704)$ |
| Cluster 3 | $197 / 197$ | $458 / 500$ | $264 / 418$ | $191 / 251$ | $846 / 1366(0.6193)$ |
| Cluster 4 | $179 / 191$ | $373 / 454$ | $111 / 245$ | $163 / 258$ | $552 / 1148(0.4808)$ |

Table 10. Wins and total matches for different cluster pairings, 2012

|  | Cluster 1 | Cluster 2 | Cluster 3 | Weaker opponent |
| :--- | :---: | :---: | :---: | :---: |
| Cluster 1 | $507 / 807$ | $89 / 374$ | $272 / 567$ | $0 / 1748(0.0000)$ |
| Cluster 2 | $569 / 607$ | $622 / 967$ | $518 / 578$ | $1087 / 2152(0.5051)$ |
| Cluster 3 | $435 / 611$ | $119 / 381$ | $358 / 572$ | $435 / 1564(0.2781)$ |

Table 11. Wins and total matches for different cluster pairings, 2013


[^0]:    ${ }^{1}$ For details for Weissbock's work, we direct the reader to [12].

[^1]:    ${ }^{2}$ The choice of seasons is due to presentation concerns, especially in the case of visualizations. Other/additional seasons exhibit similar phenomena.

[^2]:    ${ }^{3}$ Other seasons show similar behavior, so we treat 2008 as a representative example.

[^3]:    ${ }^{4}$ The results for other seasons can be found in Appendix A.

[^4]:    ${ }^{5}$ Although some might be similar enough to be merged.

