Mining Convex Polygon Patterns

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Introduction (1)

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$\mathcal{G}$	i_1	i_2	i_3	*class*
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b		X	X	+
c		X	X	−
d	X	X	X	+
e	X		X	−
f		X		−

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Hypothesis.

When objects have items i_2 and i_3
The + class is fostered

Relative Accuracy*: Difference between the proportion of the class under the hypothesis and its proportion in the whole dataset.

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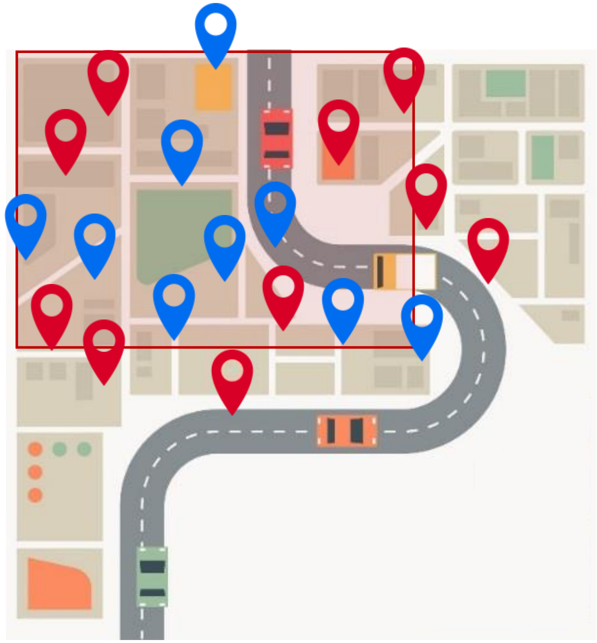

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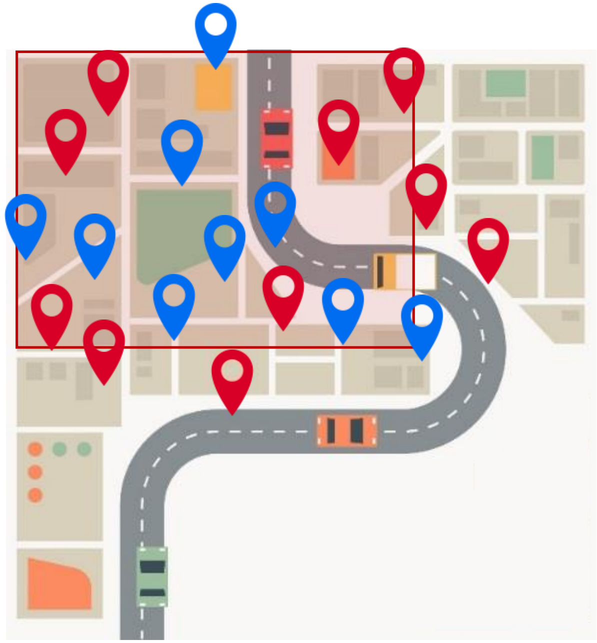

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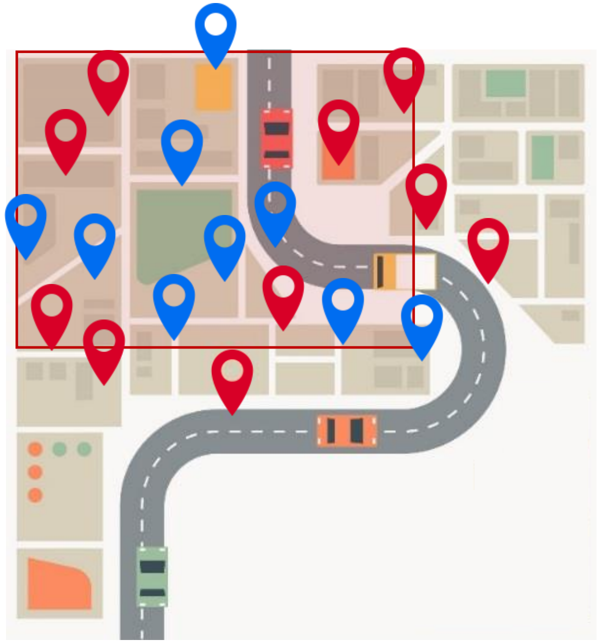

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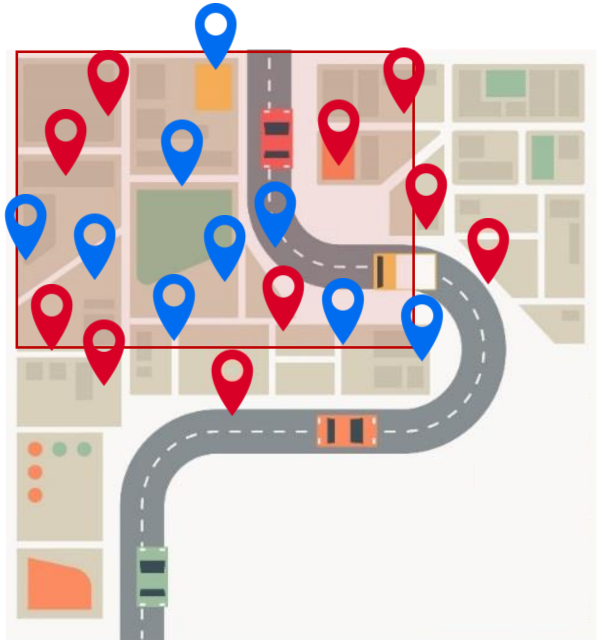

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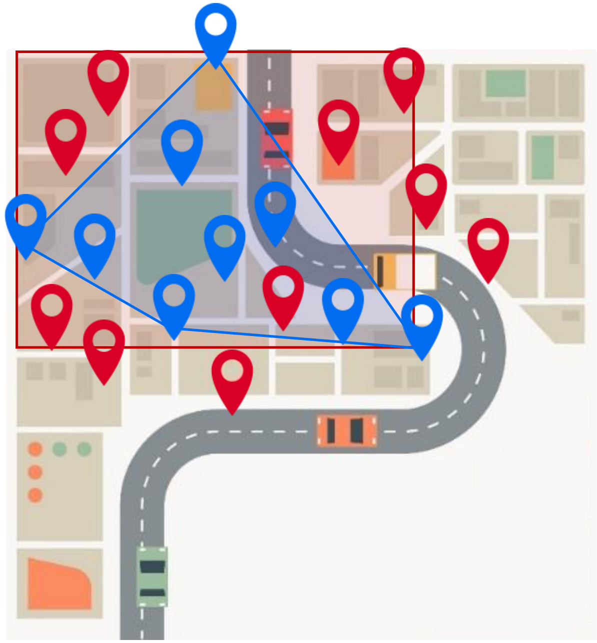
- We need to have more expressive patterns without overfitting the data.
- A good trade-off is the use of convex polygon patterns.



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Interval Pattern

$\mathcal{G}$	x	y
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d	3	4
e	4	3
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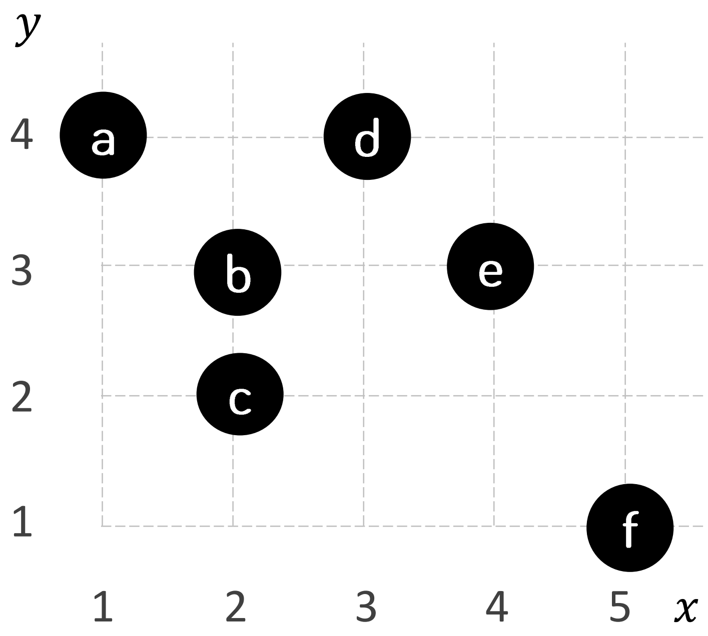
Two numerical attributes x and y



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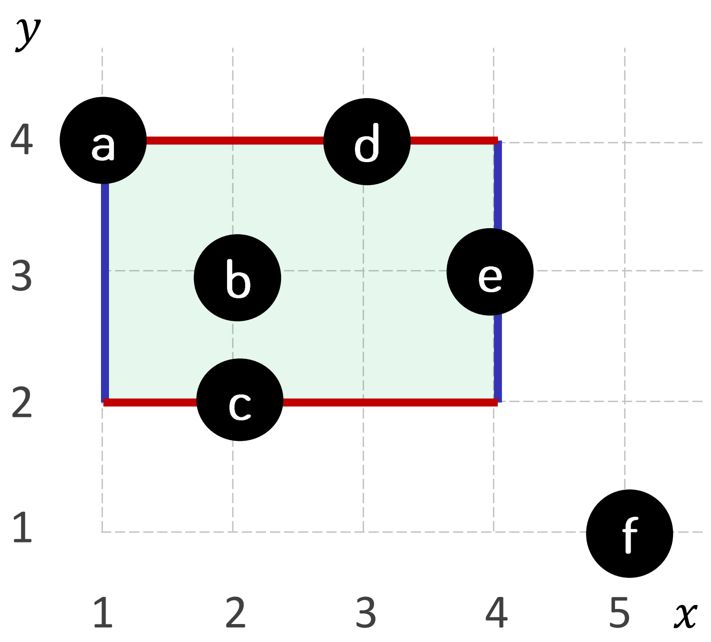

Intent. $1 \leq x \leq 4$ ***AND*** $2 \leq y \leq 4$.

Extent. $\{a, b, c, d, e\}$

Interval Pattern Language [Kaytoue, M. et al. (IJCAI, 2011)]

- An intent is a rectangle (interval restriction over each attribute).

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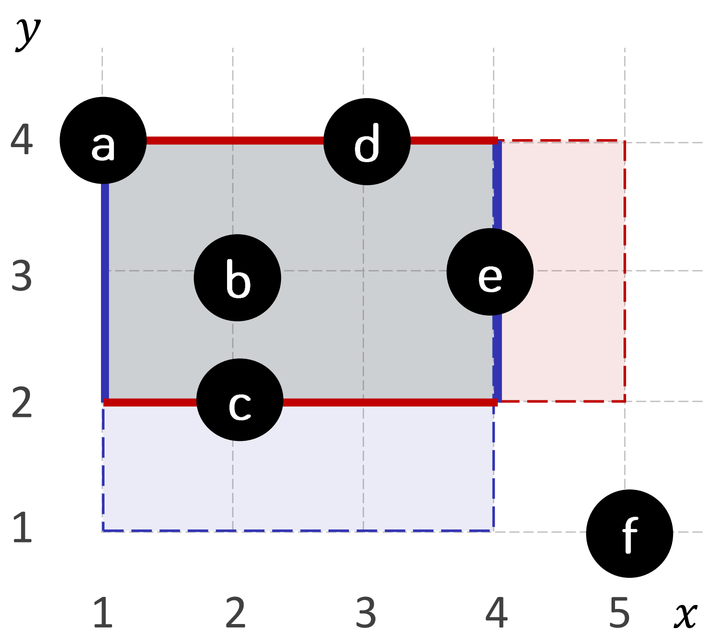

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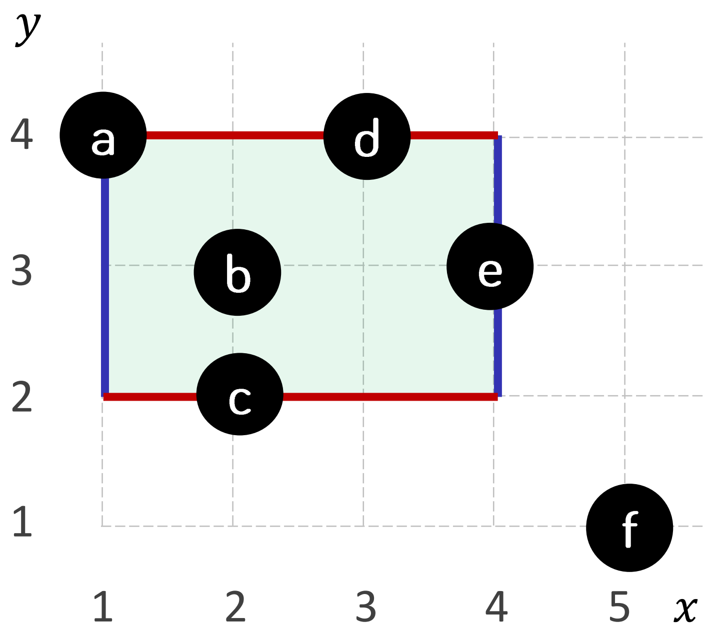

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- An intent is a rectangle (interval restriction over each attribute).
- Same set of objects can be described by different intents.
- We are interested in the most restrictive ones (i.e. tightest ones, closed ones).

Convex Polygon Pattern

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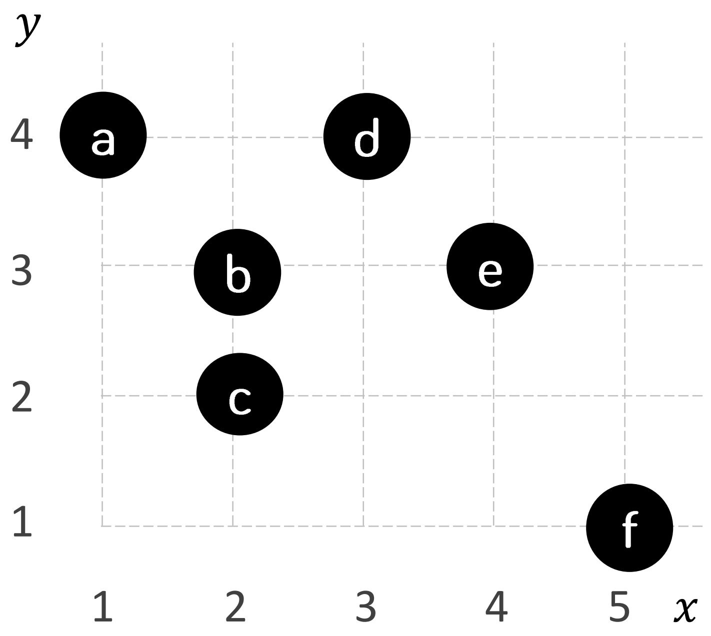

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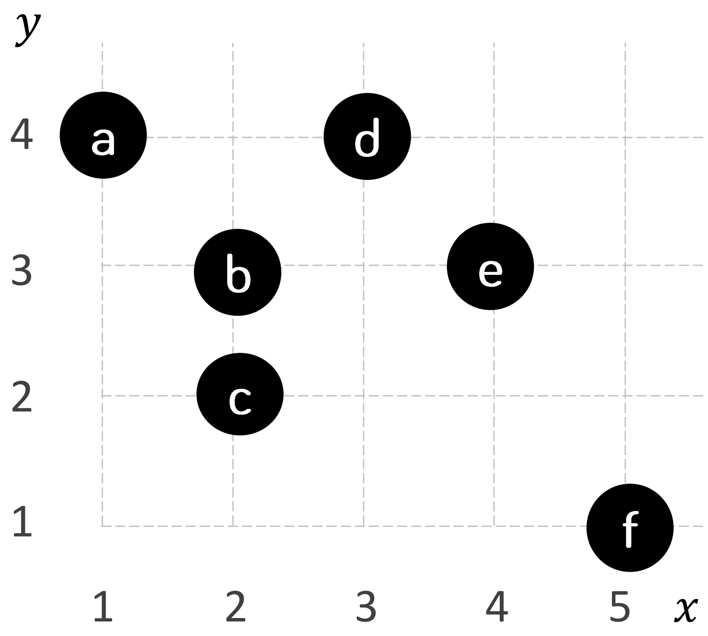

The Proposed Pattern Language.

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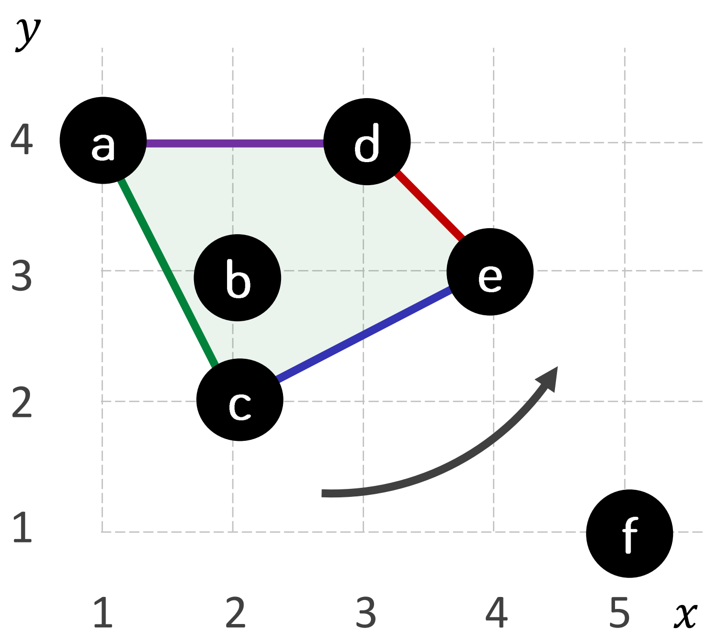

Intent.

$$y \leq 4 \quad AND$$
$$x + y \leq 7 \quad AND$$
$$x - 2y \leq -2 \quad AND$$
$$2x + y \geq 6 \quad .$$

(H-representation)

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- An intent is a convex polygon (conjunction of linear inequalities).

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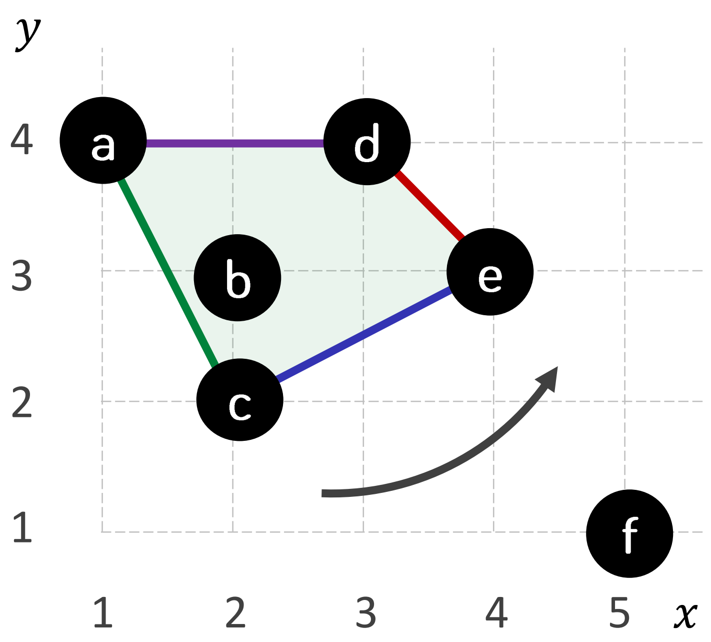

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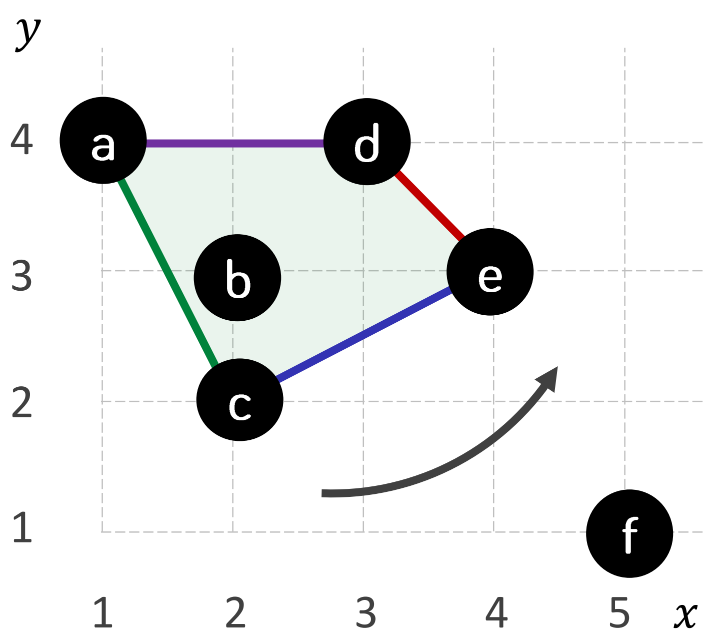

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- An intent is a convex polygon (conjunction of linear inequalities).
- The most restrictive description of a set of objects is its convex hull.
- We will represent the intent as an ordered sequence of extreme points $[a, c, e, d]$ (V-representation).

Problem Definition

Let $\mathcal{G}$ be a **dataset** with one **spatial attribute**. Enumerate all possible convex hulls that can be built using point subsets of $\mathcal{G}$ without redundancy respecting eventually a set of user-specified constraints.

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1. Algorithms
2. Experiments
3. Perspective

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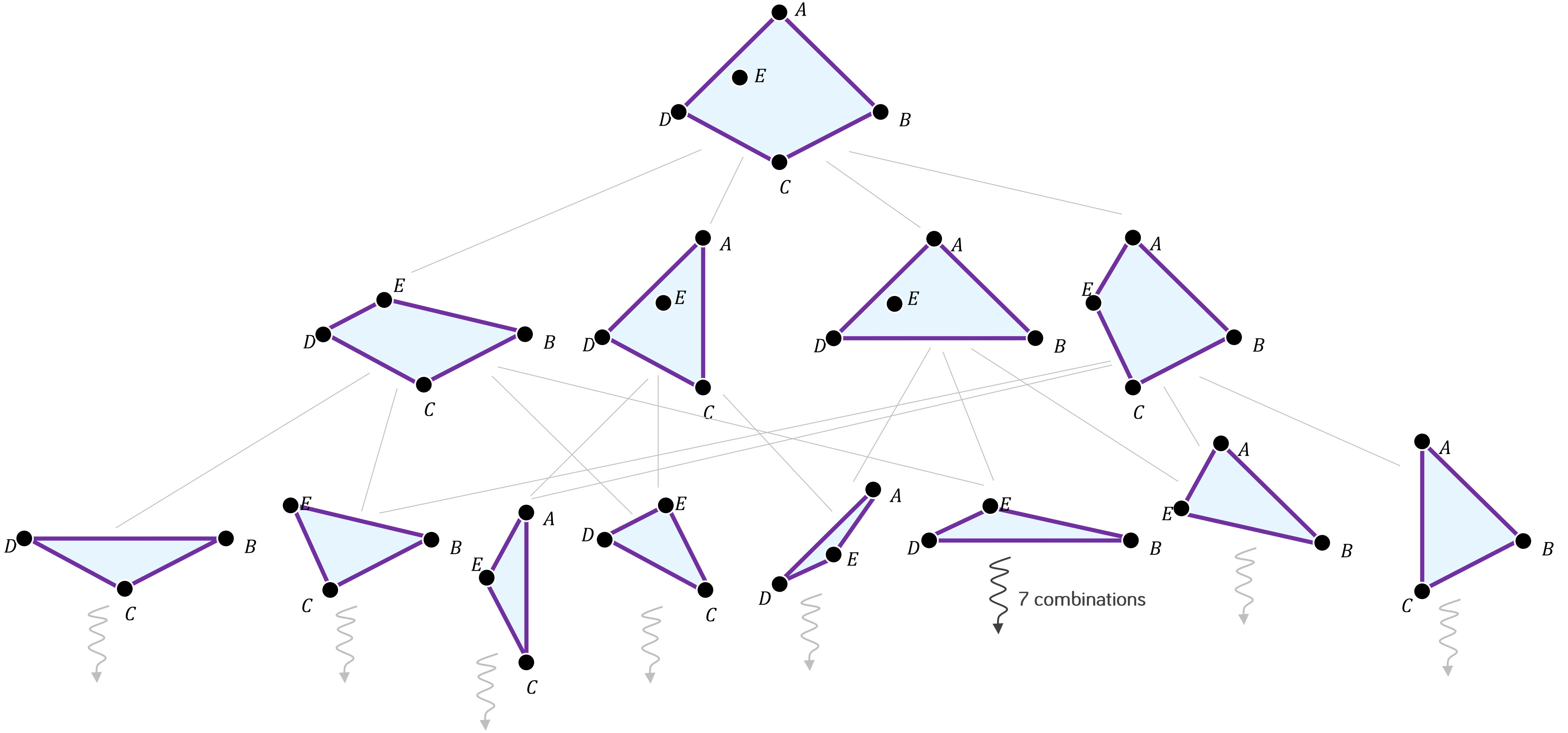
Search space – convex polygons lattice



Enumeration Algorithms - Summary

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CloseByOne

- Bottom-up enumeration.
- Compute convex hulls.
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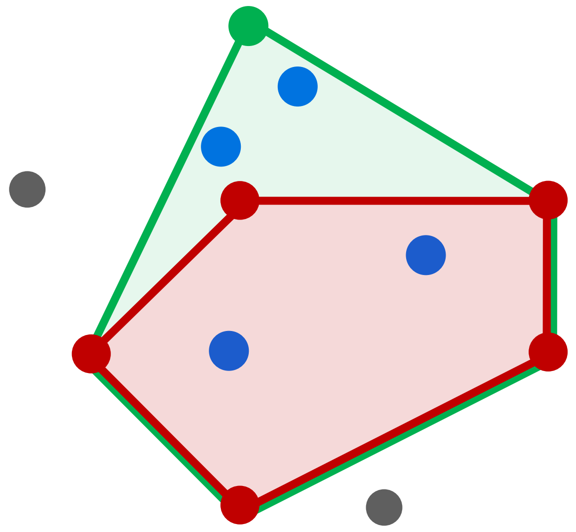

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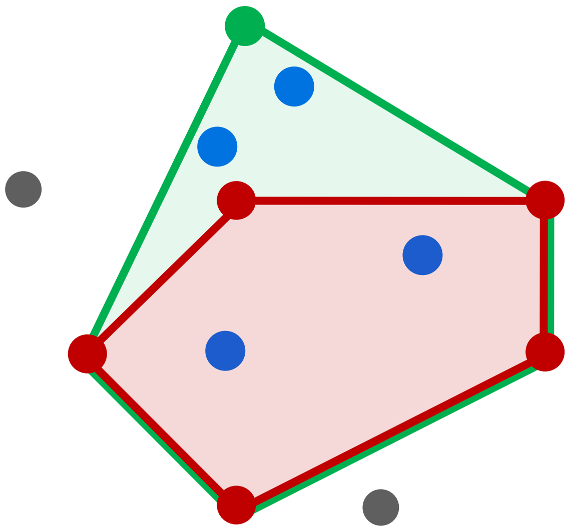

DT-Based

- Top-down enumeration.
- Based on Delaunay Triangulation.
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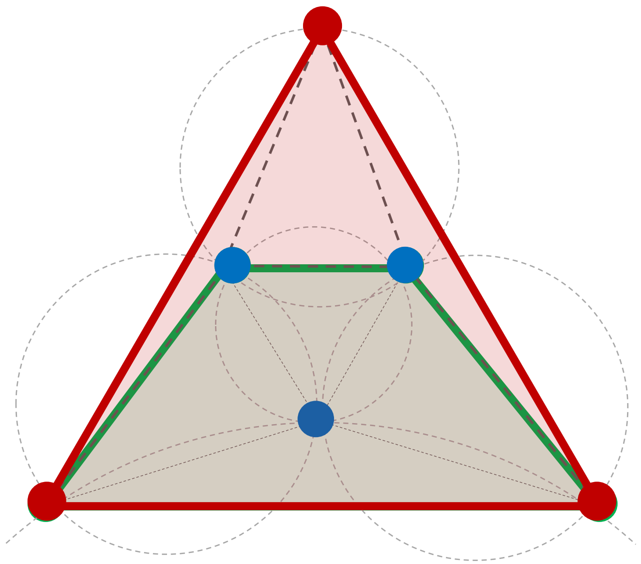

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Vision-Based

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- Number of vertices increases by one after each extension.
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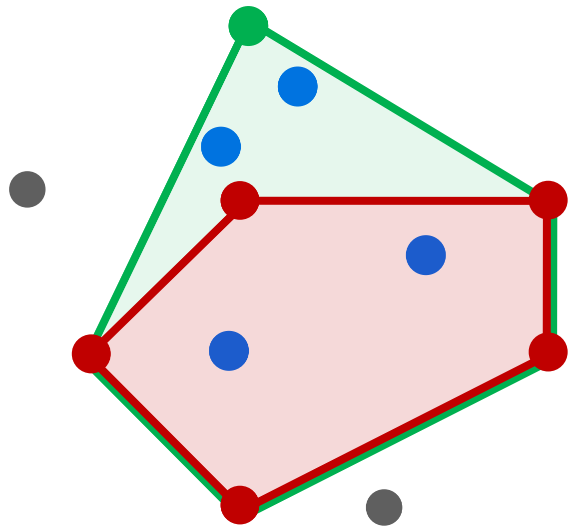
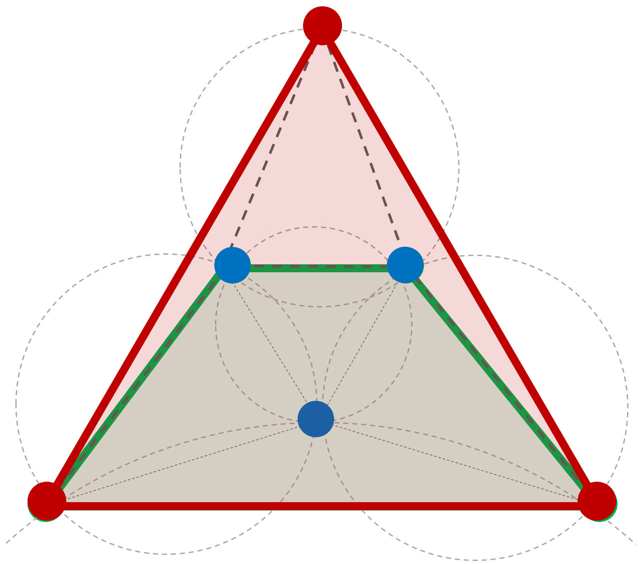
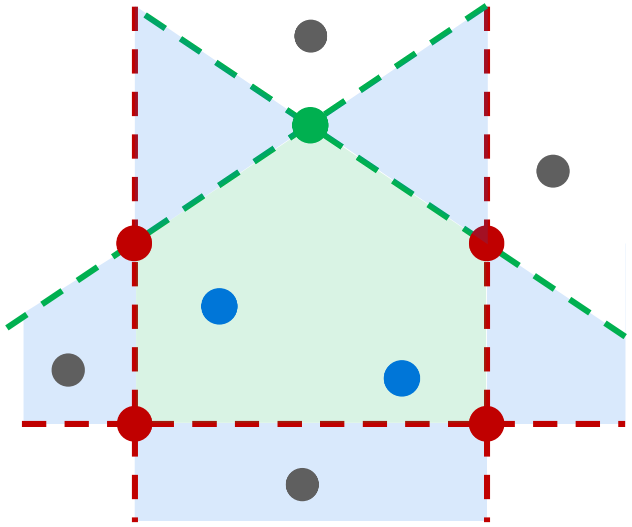
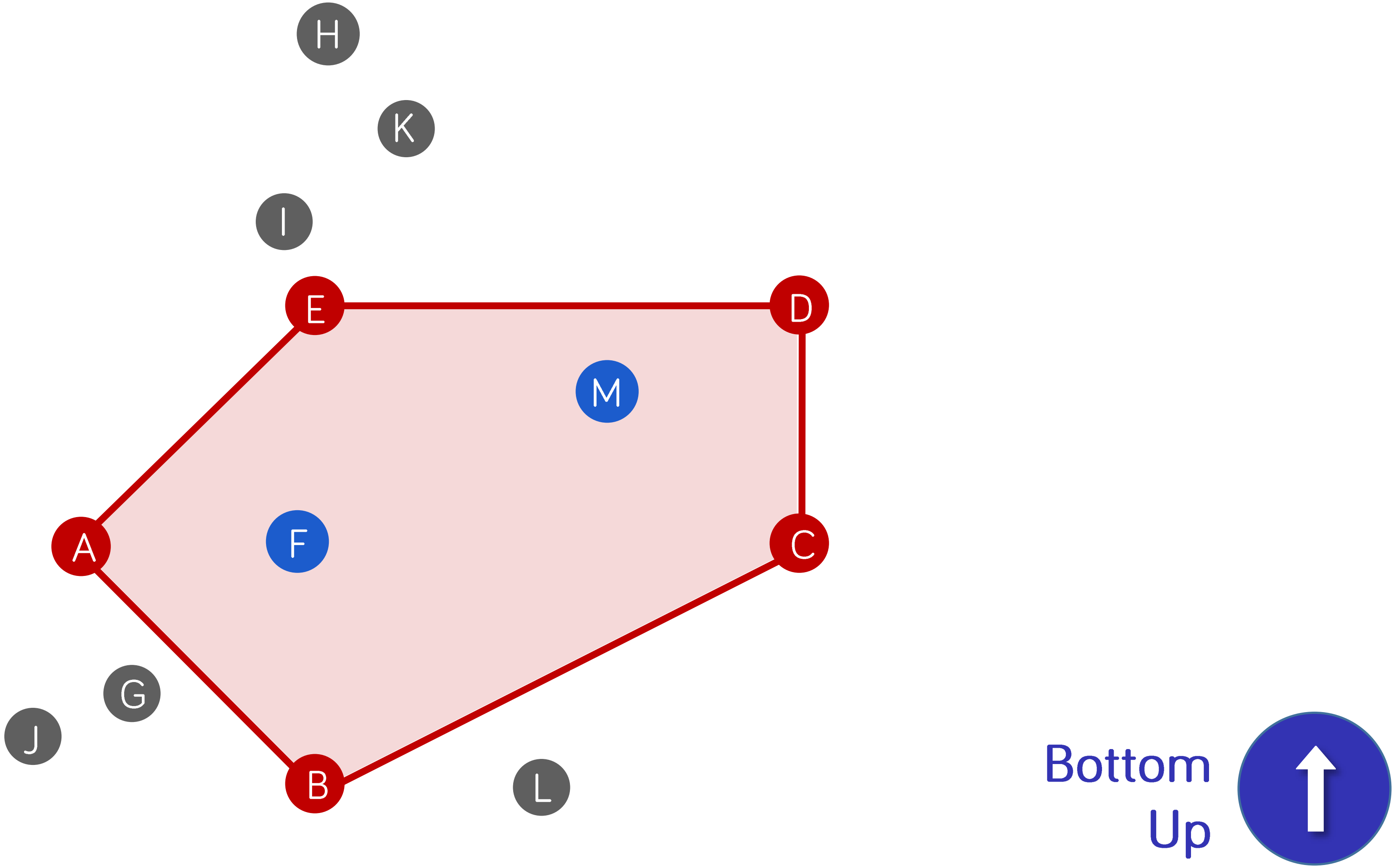






Algorithm 1 - CloseByOne

Objects in the dataset are canonically ordered ($A < B$ ...)



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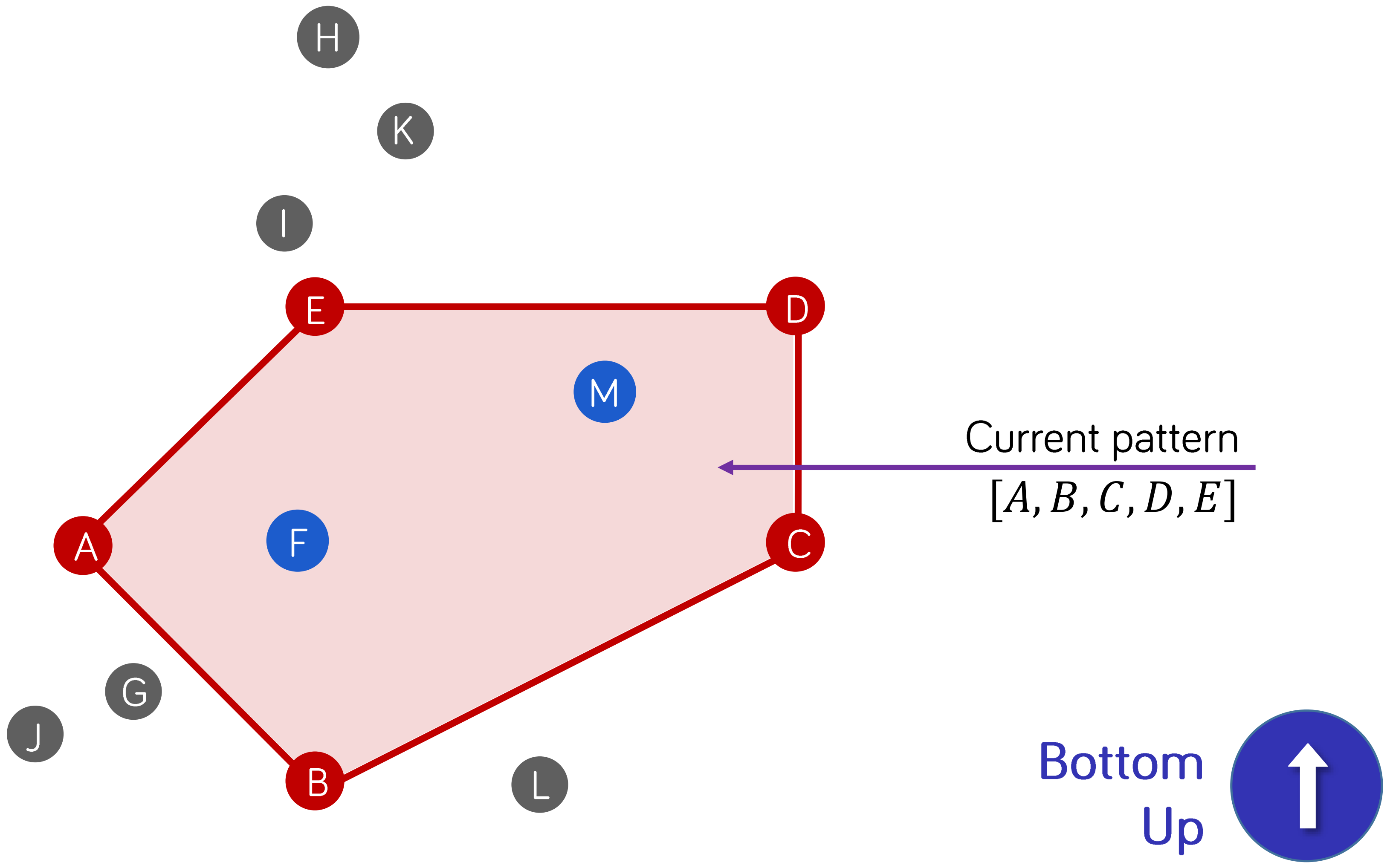

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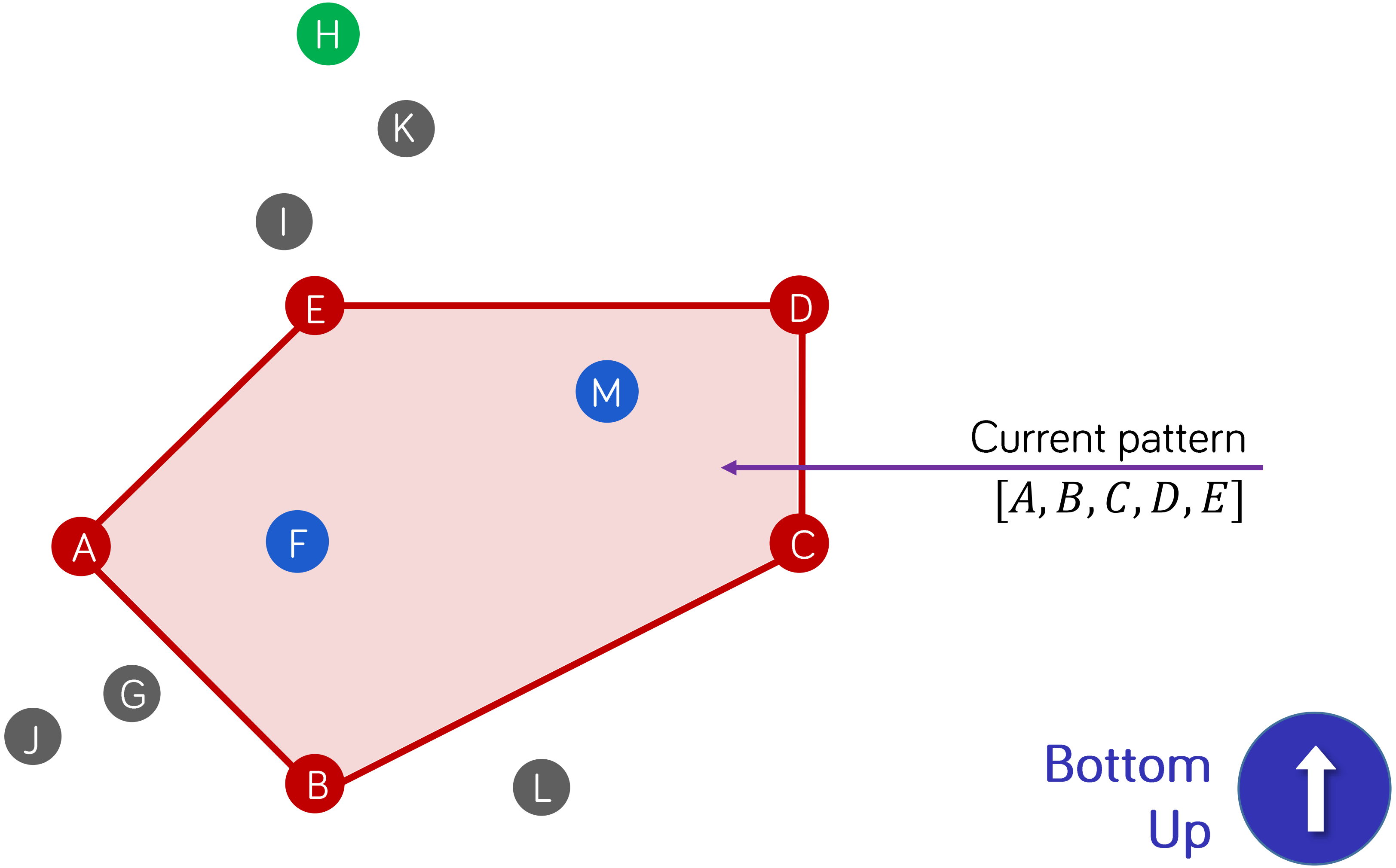

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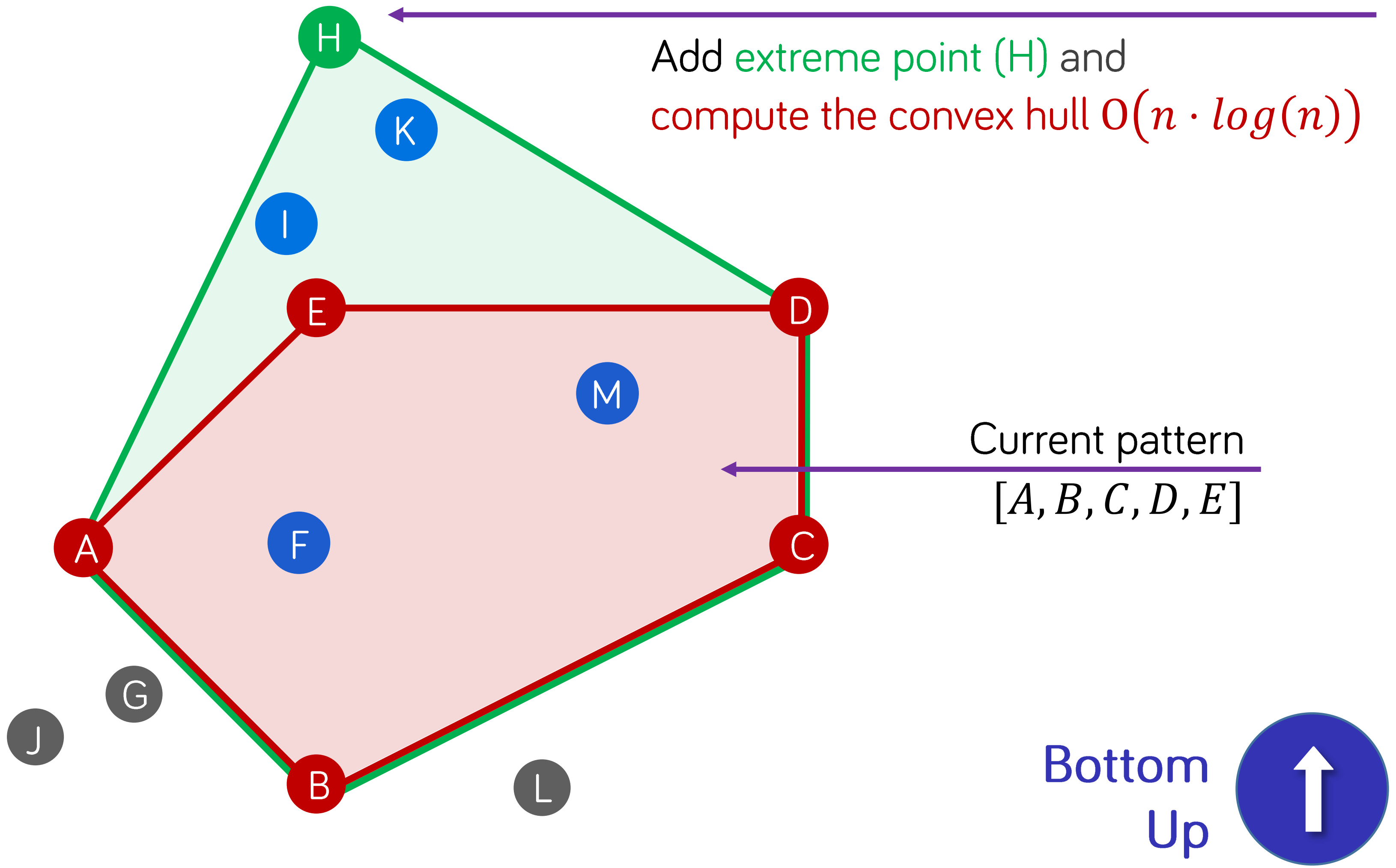

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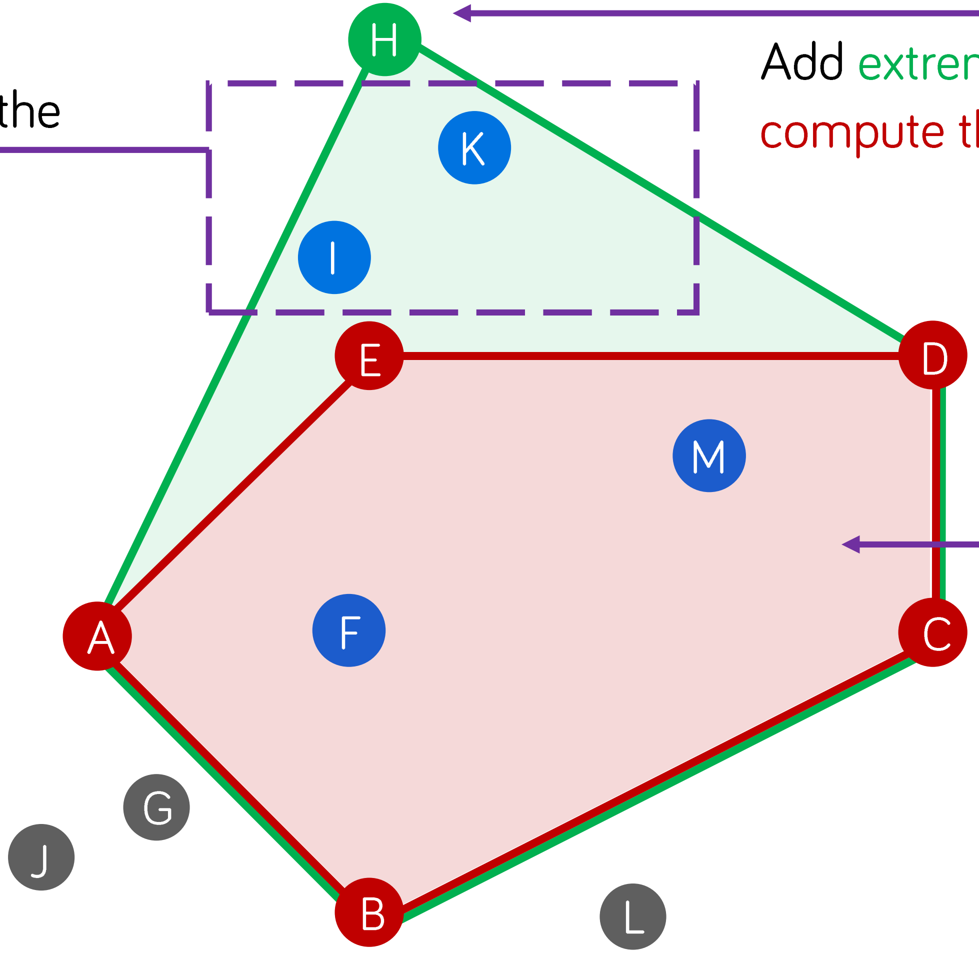
All the newly added objects to the extent comes after (I, $K > H$)



Add extreme point (H) and compute the convex hull $O(n \cdot log(n))$

Current pattern $[A, B, C, D, E]$

Bottom Up

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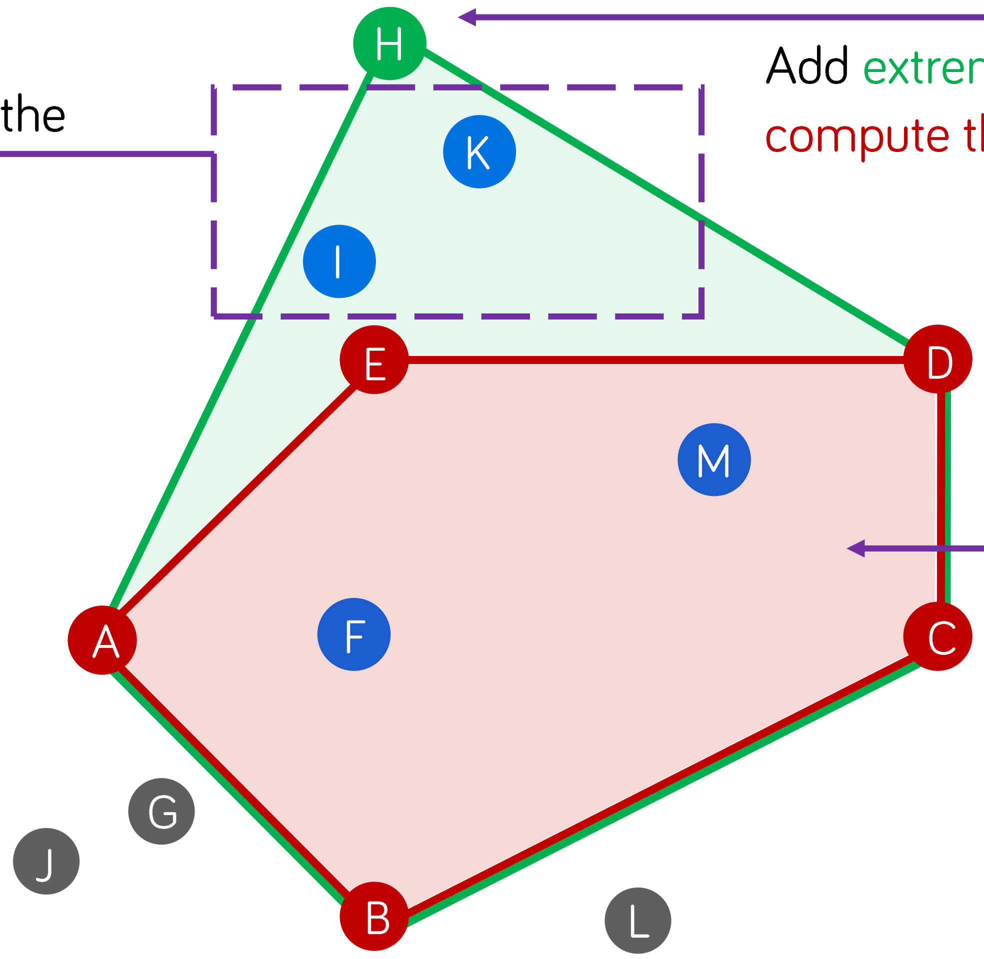

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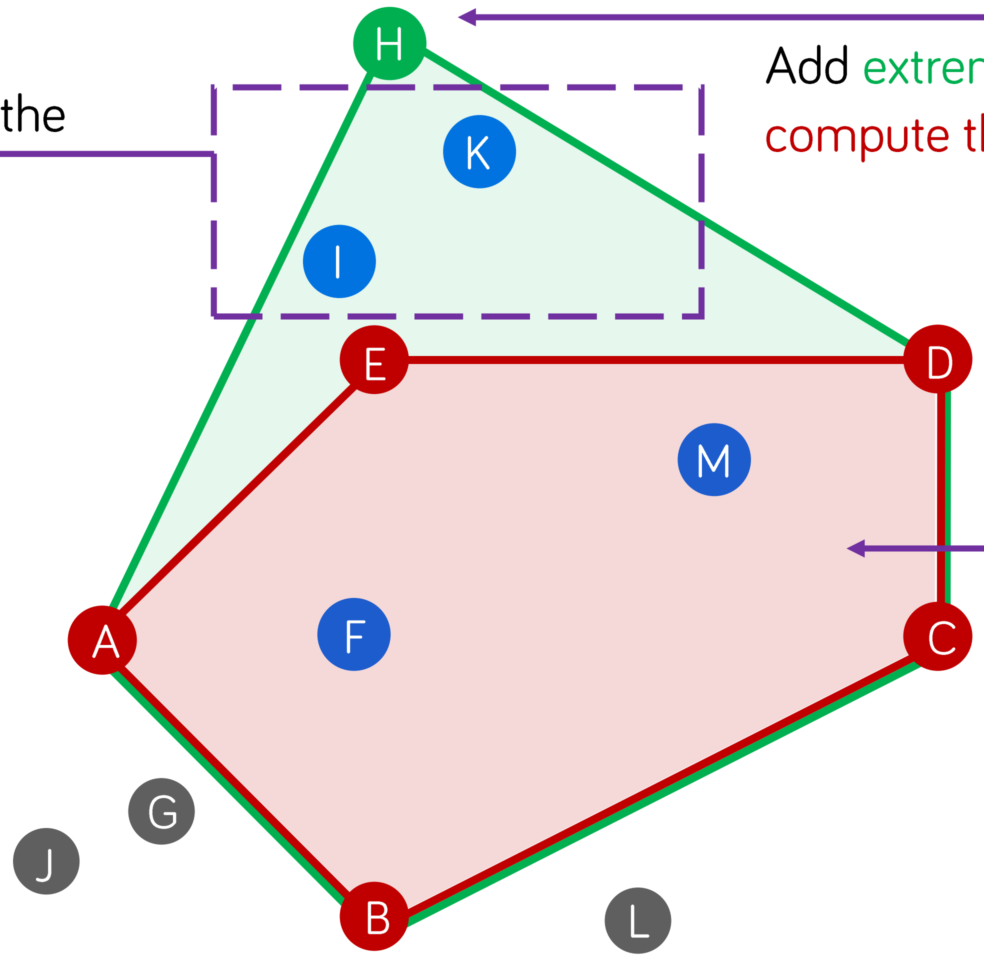

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If the pattern is already visited the algorithm backtracks.

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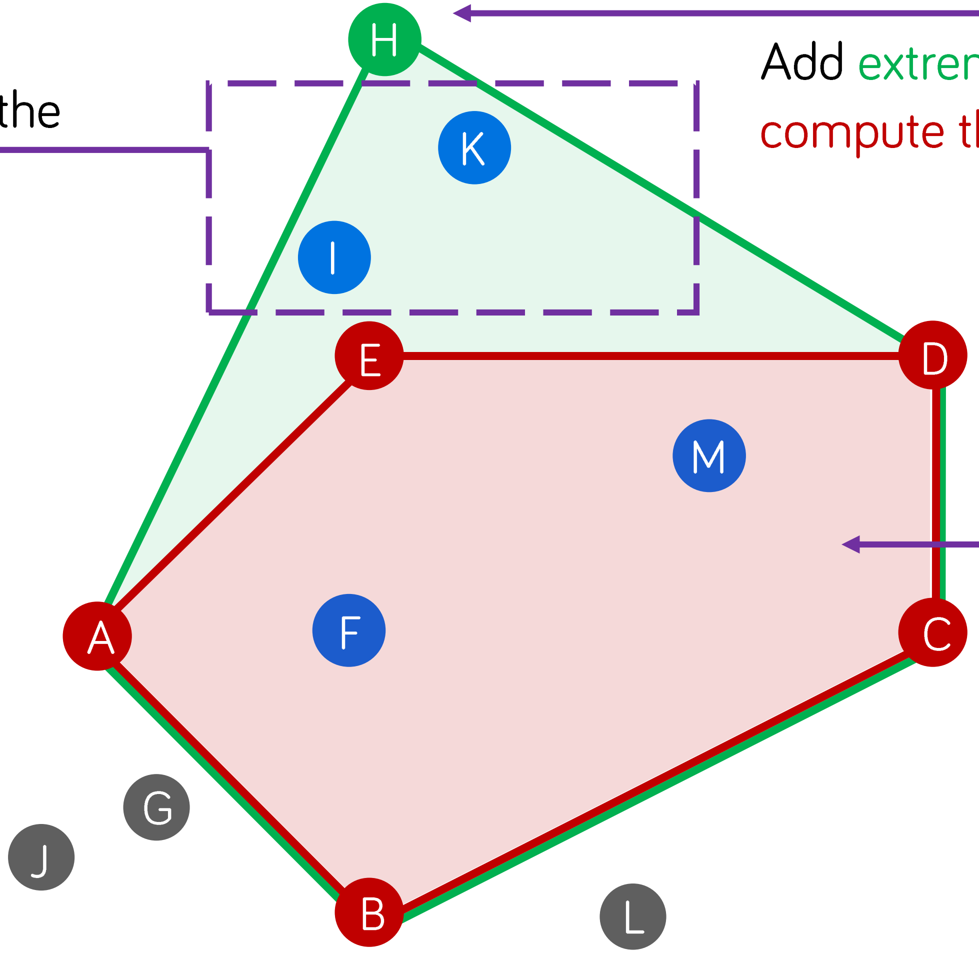

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Continue enumeration by adding extreme point ($J > H$)

Bottom Up

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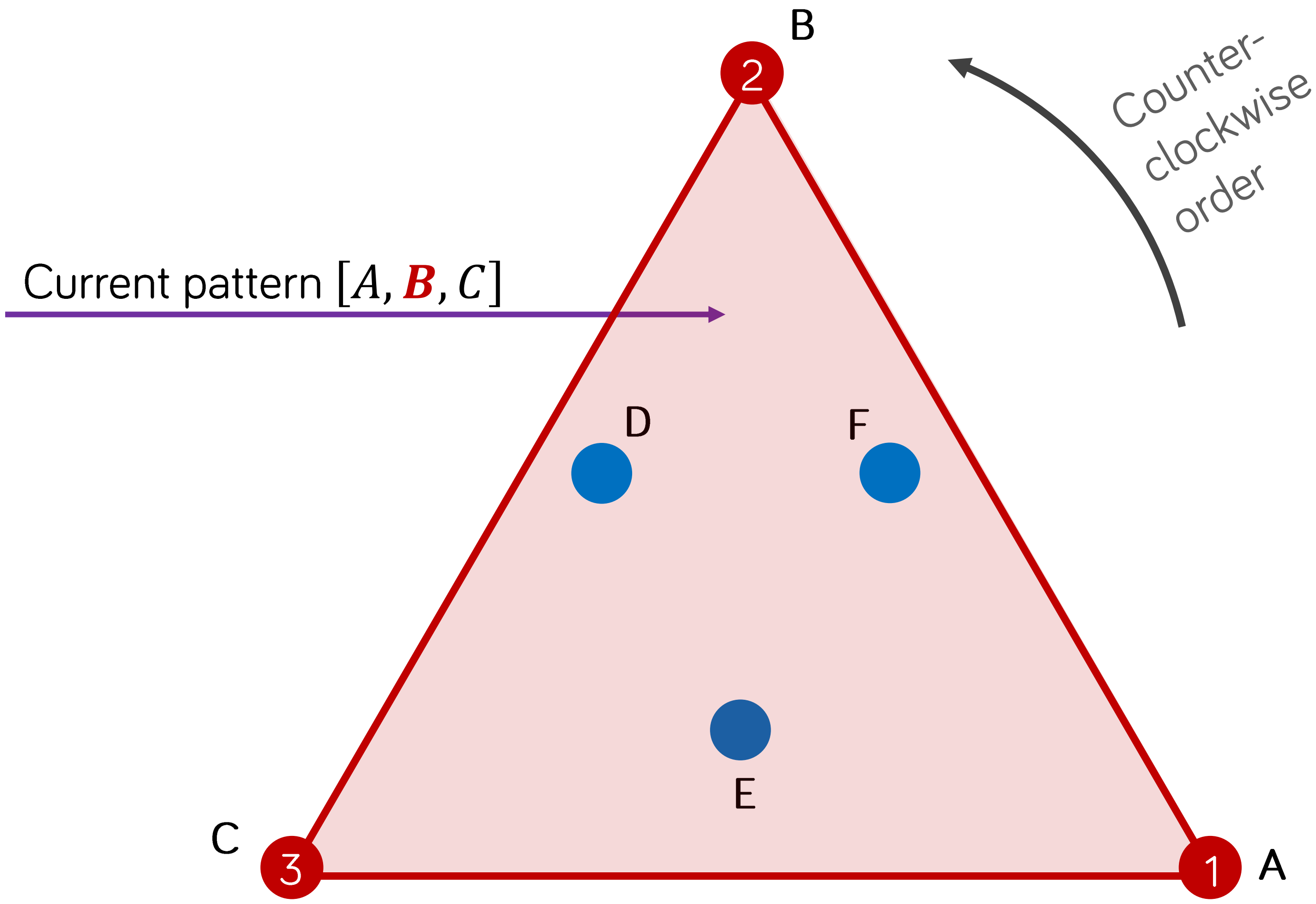
Algorithm 2 – Delaunay Triangulation Based



Top Down

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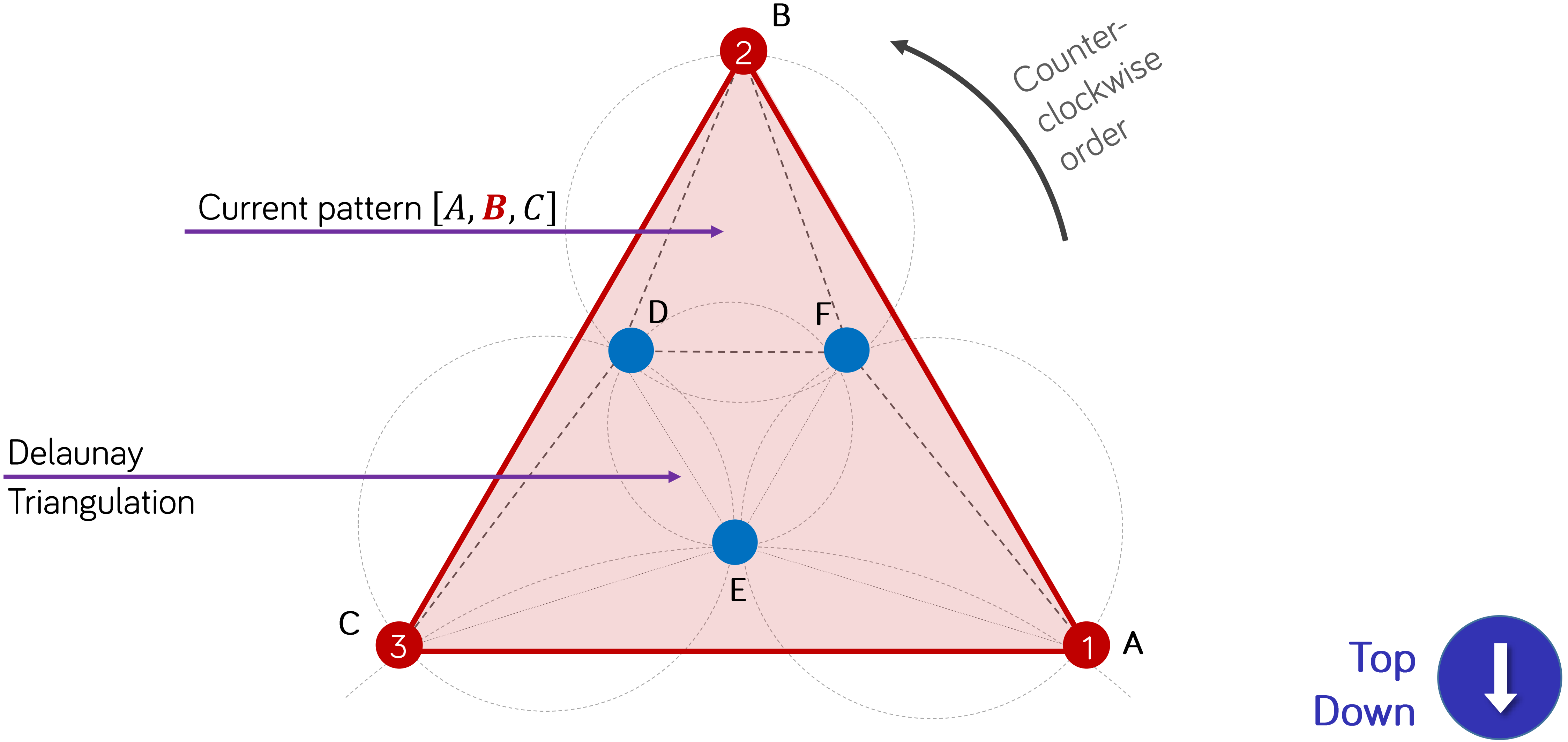

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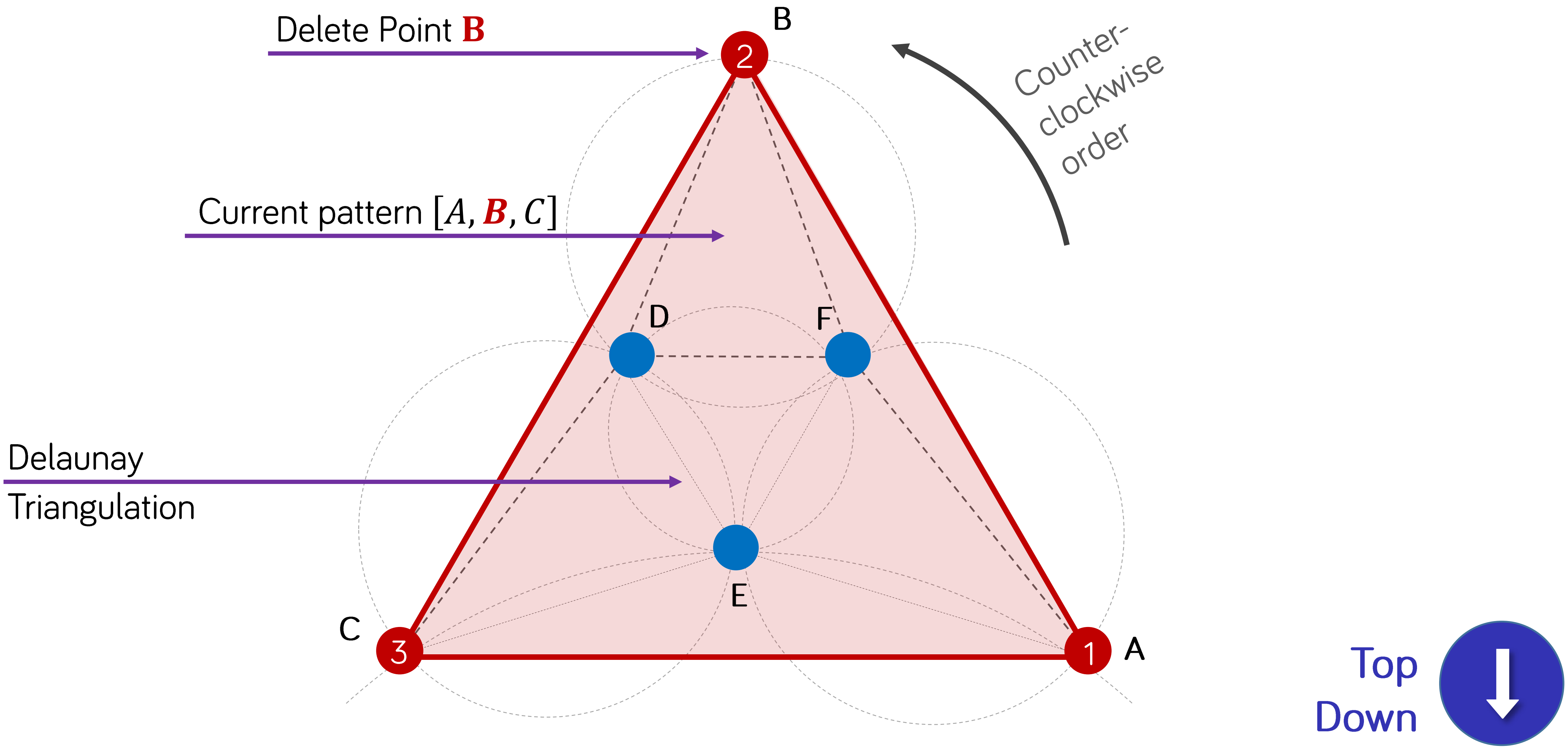

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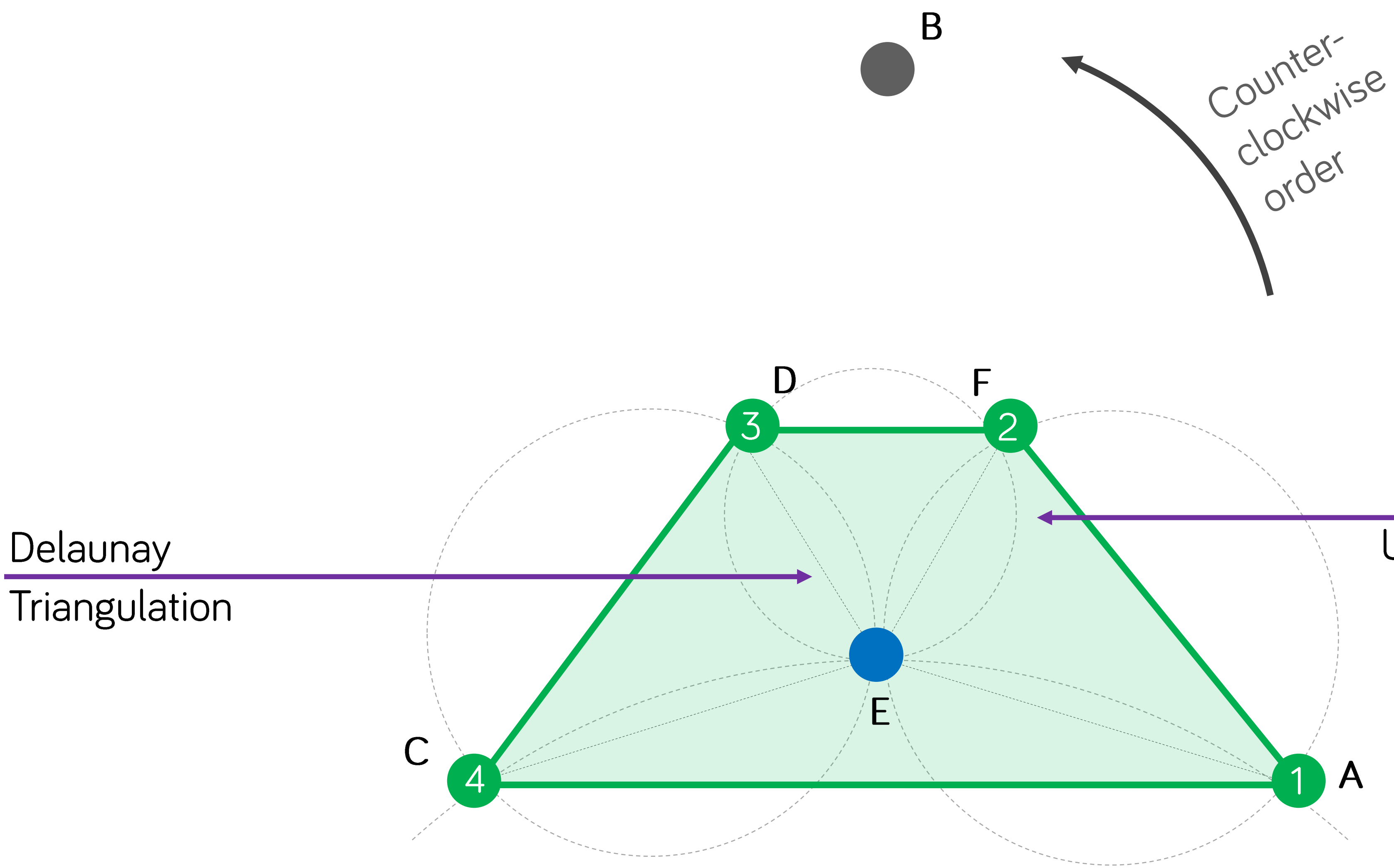

New pattern $[A, \mathbf{F}, \mathbf{D}, C]$

Update Delaunay Triangulation

$O(k \cdot log(k))$

Top Down

*k designates the number of neighbors

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Continue enumeration by deleting F (position 2)

Delaunay Triangulation

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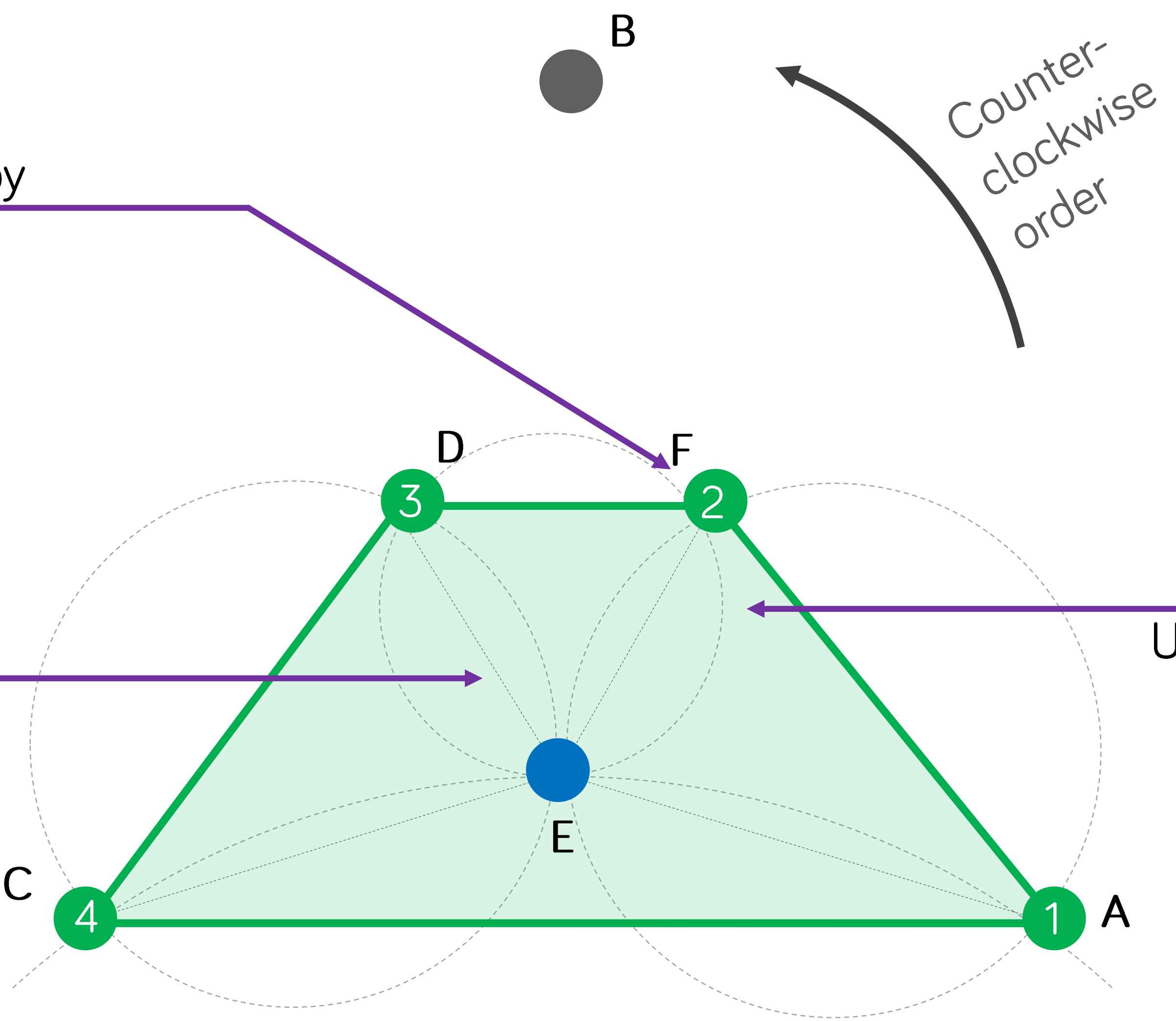

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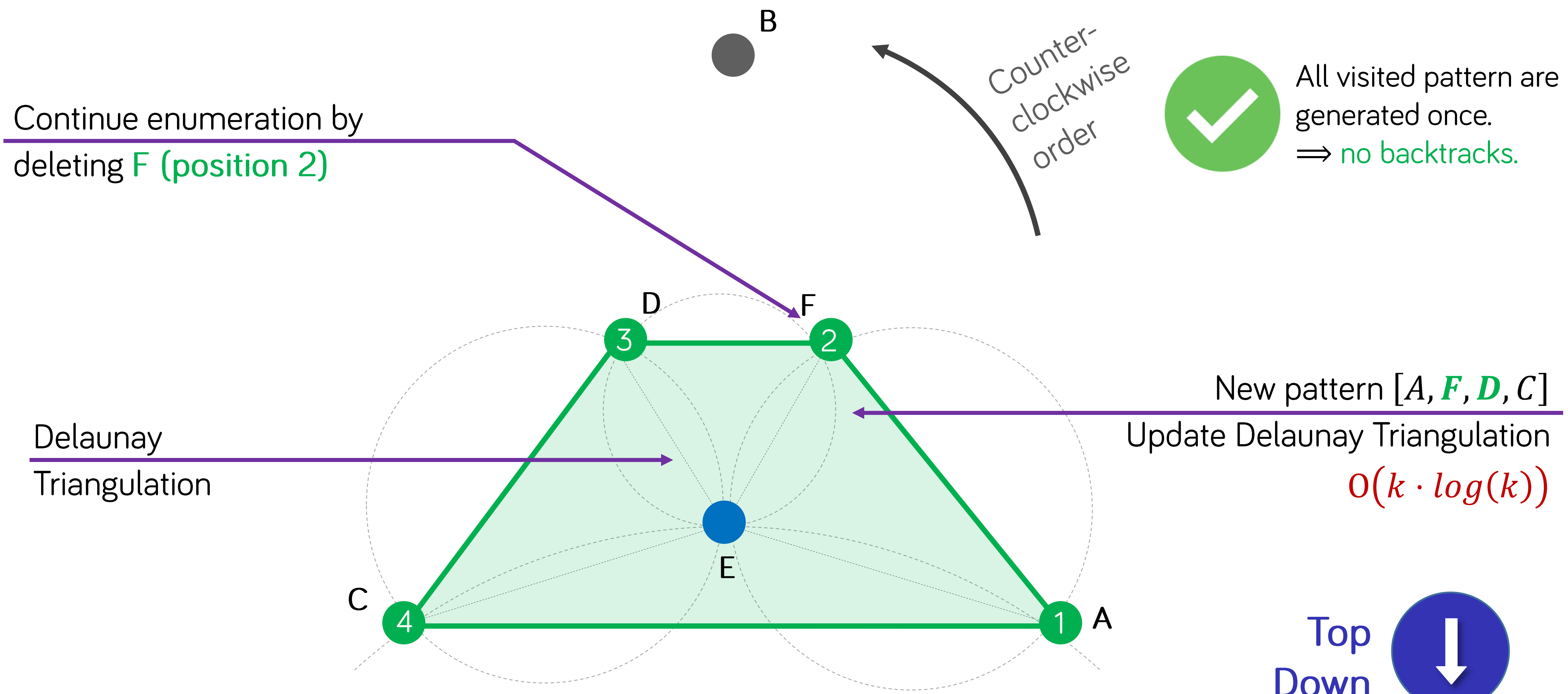

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Let be $E \subset \mathbb{R}^2$ and $e \in \mathbb{R}^2$. Let be $\boldsymbol{ch(E)}$ the set of vertices of the convex hulls of E. We have:

$$|\boldsymbol{ch(E \cup \{e\})}| \leq |\boldsymbol{ch(E)}| + 1$$

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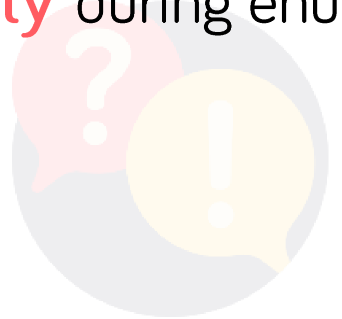

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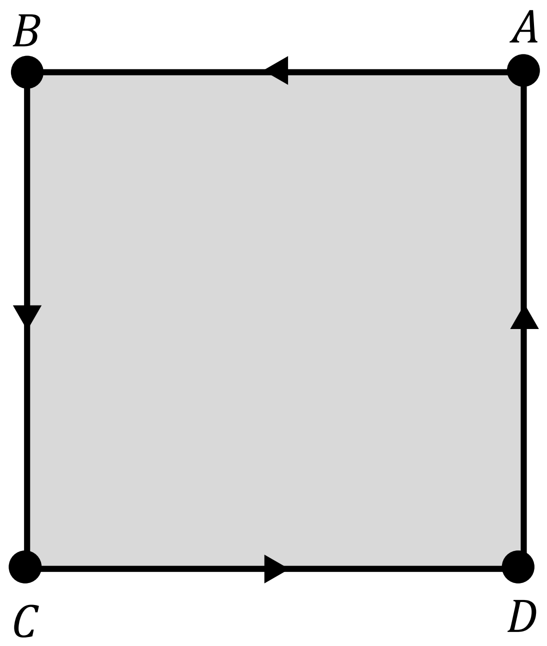
The constraint "maximum number of vertices $\leq \tau$ " becomes monotonic.

Algorithm 3 – Vision Based (2)



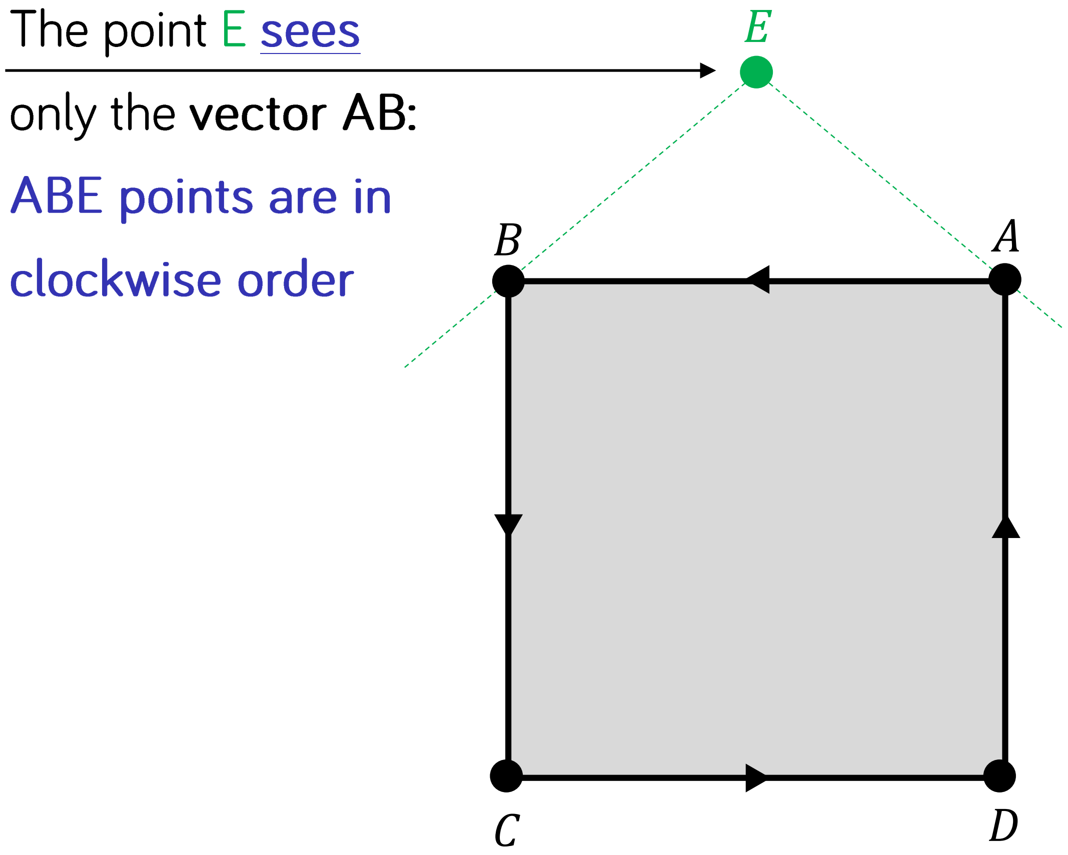
Algorithm 3 – Vision Based (2)

The point E sees

only the **vector** AB:

ABE points are in

clockwise order



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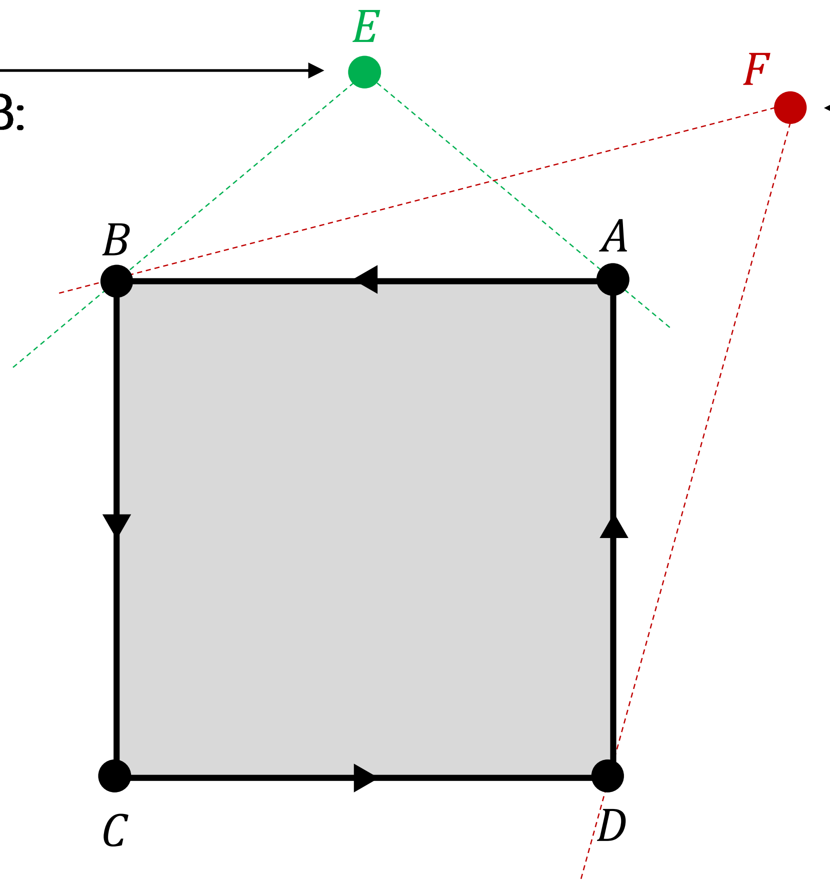

The point F sees the **vectors DA and AB**

ABF points are in clockwise order

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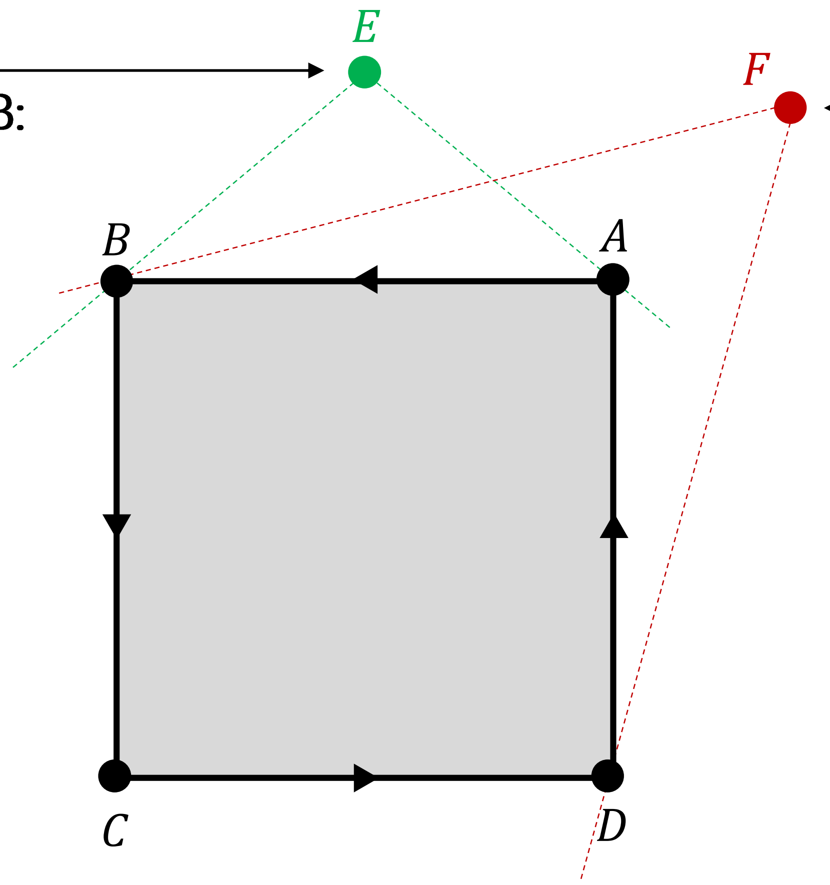

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Adding F as an **extreme point** to the description $d = [A, B, C, D]$ will destroy the extreme point A creating $d' = [F, B, C, D]$

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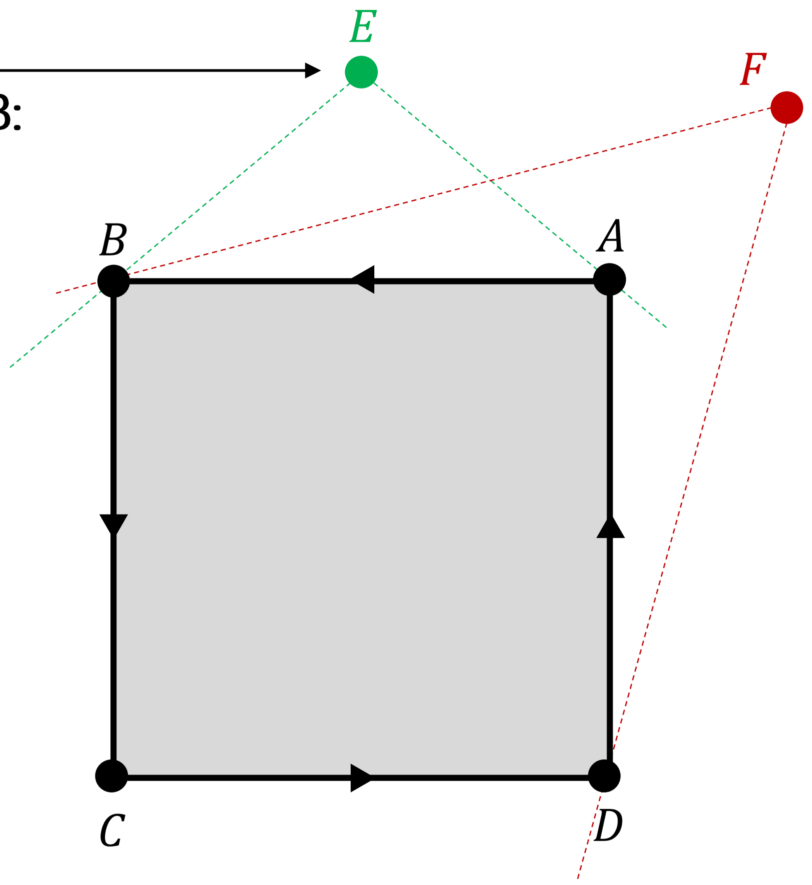

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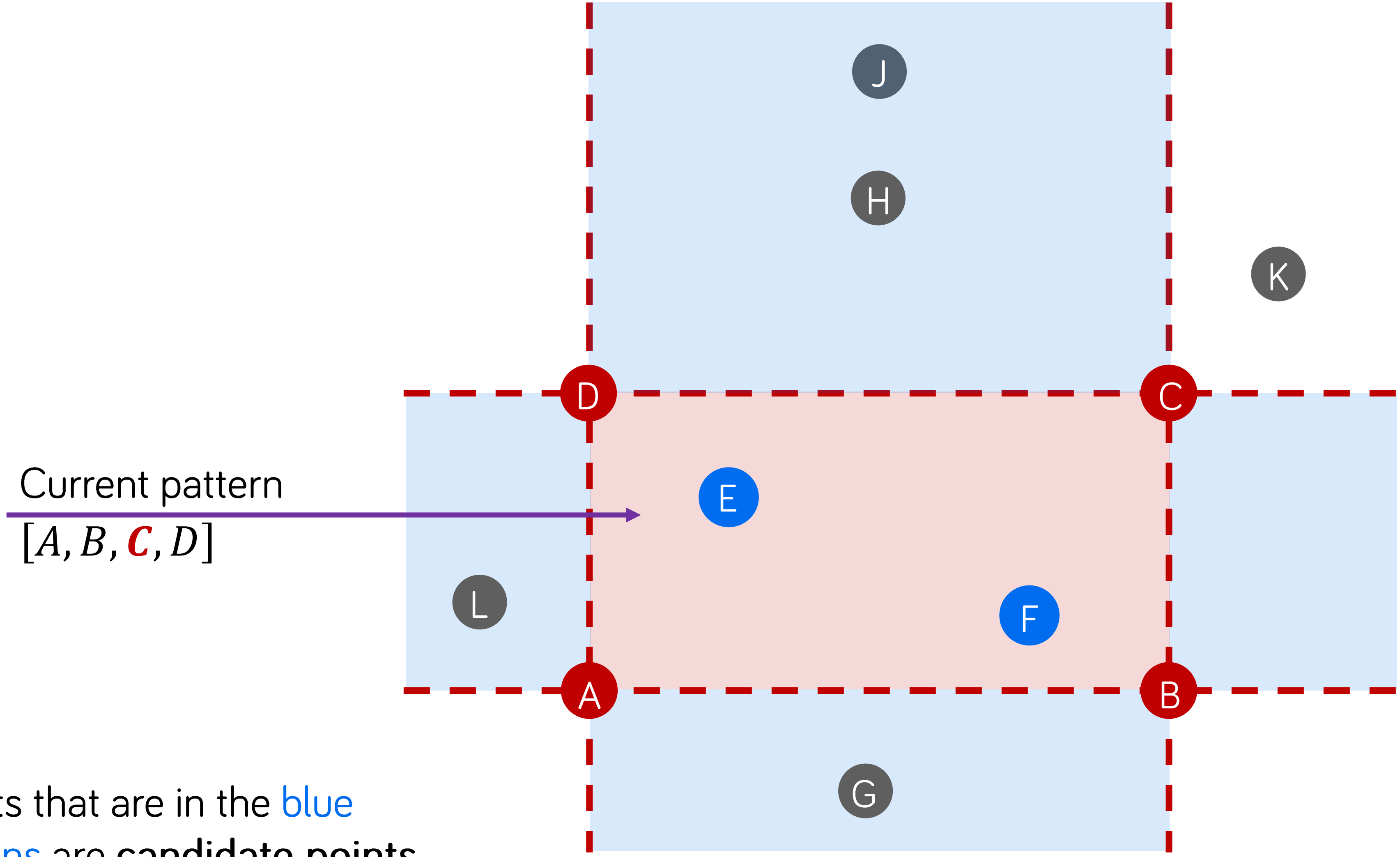
Adding F as an **extreme point** to the description $d = [A, B, C, D]$ will destroy the extreme point A creating $d' = [F, B, C, D]$

To ensure that $|\boldsymbol{d}'| = |\boldsymbol{d}| + \mathbf{1}$. One should add only points which sees only one segment. An addable points for a segment is called a **candidate point.**

Algorithm 3 – Vision Based (3)

Objects in the dataset are canonically ordered ($\boldsymbol{A} < \boldsymbol{B}$ ...)



- Points that are in the blue regions are candidate points.

Bottom Up

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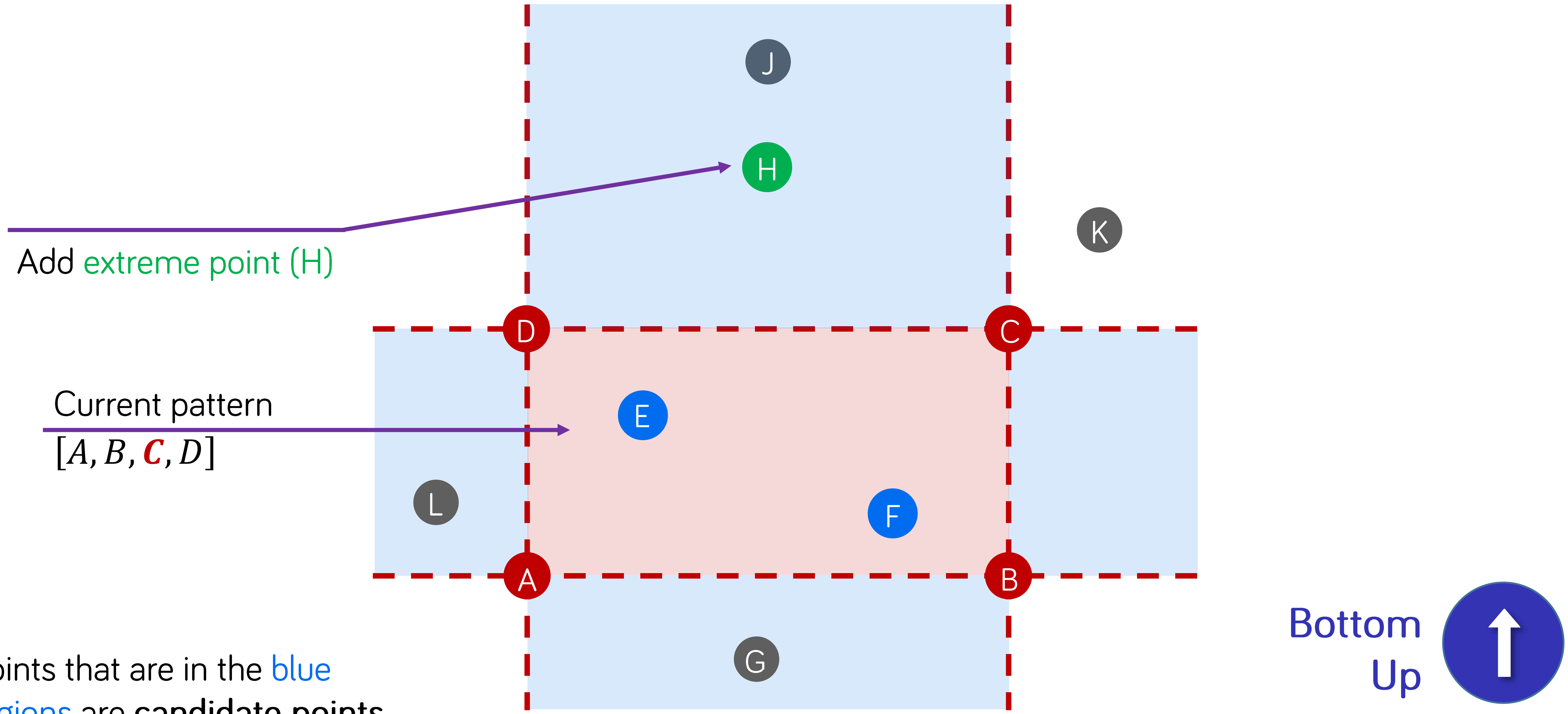

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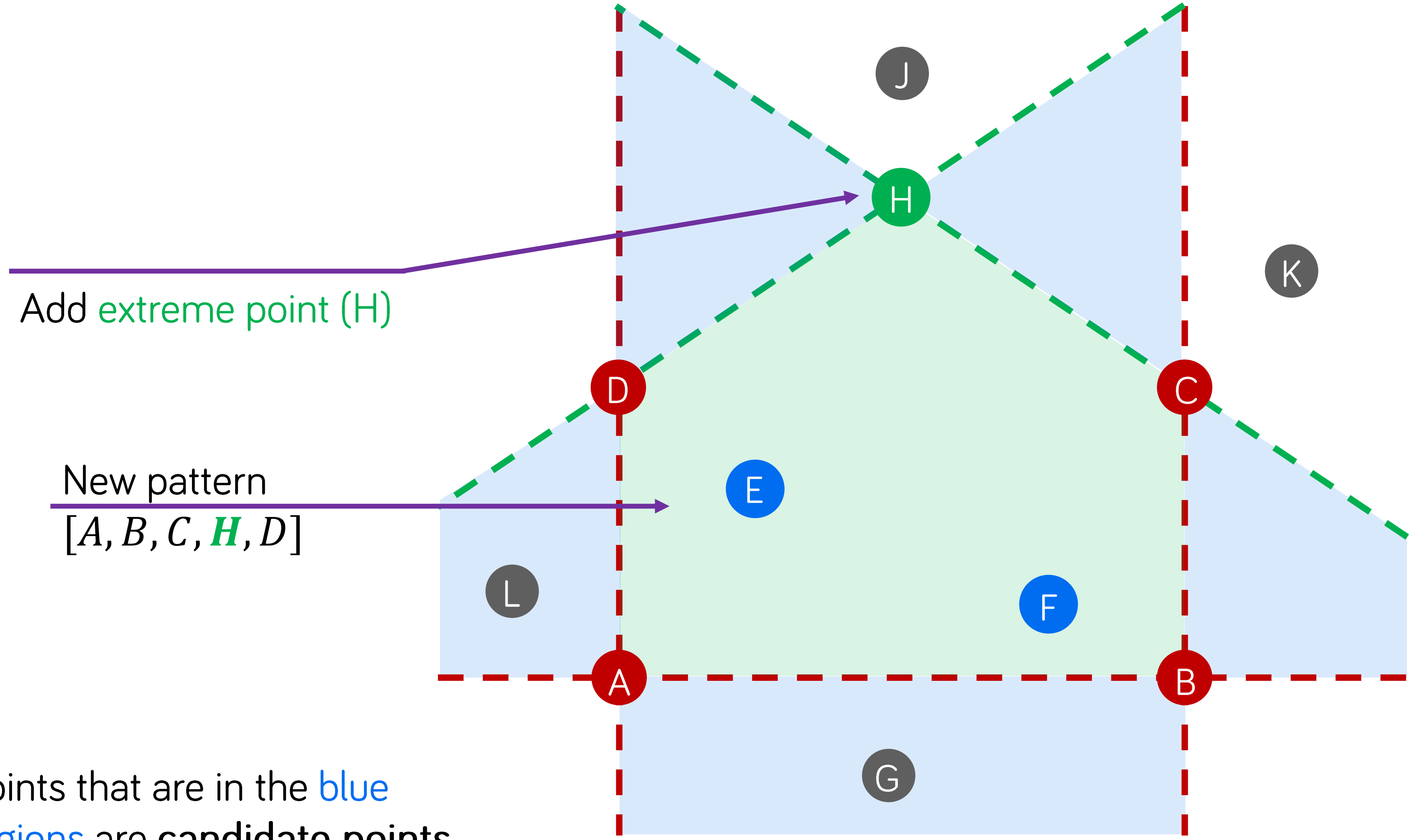

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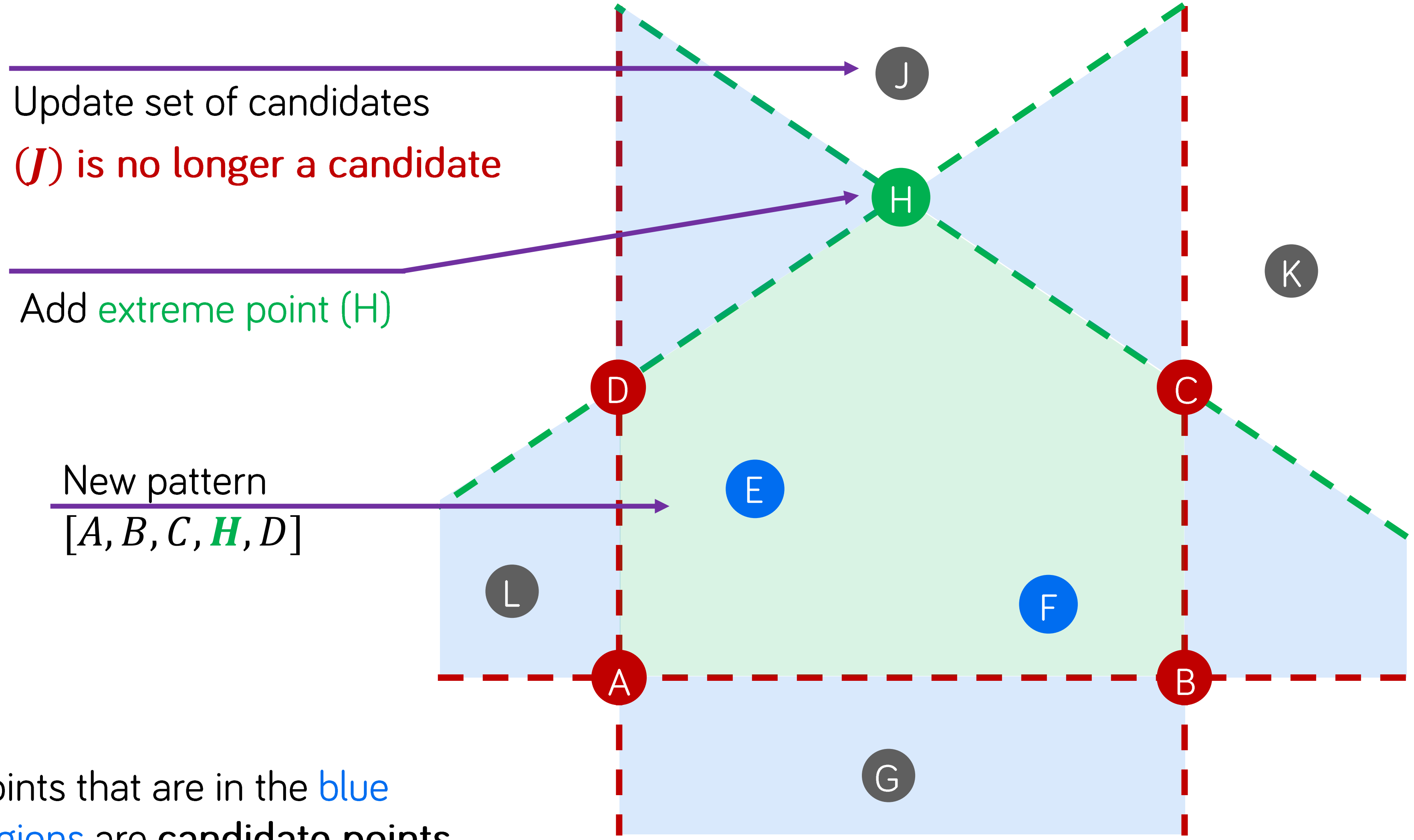

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Bottom Up

Algorithm 3 – Vision Based (3)

Objects in the dataset are canonically ordered ($\boldsymbol{A} < \boldsymbol{B}$ ...)

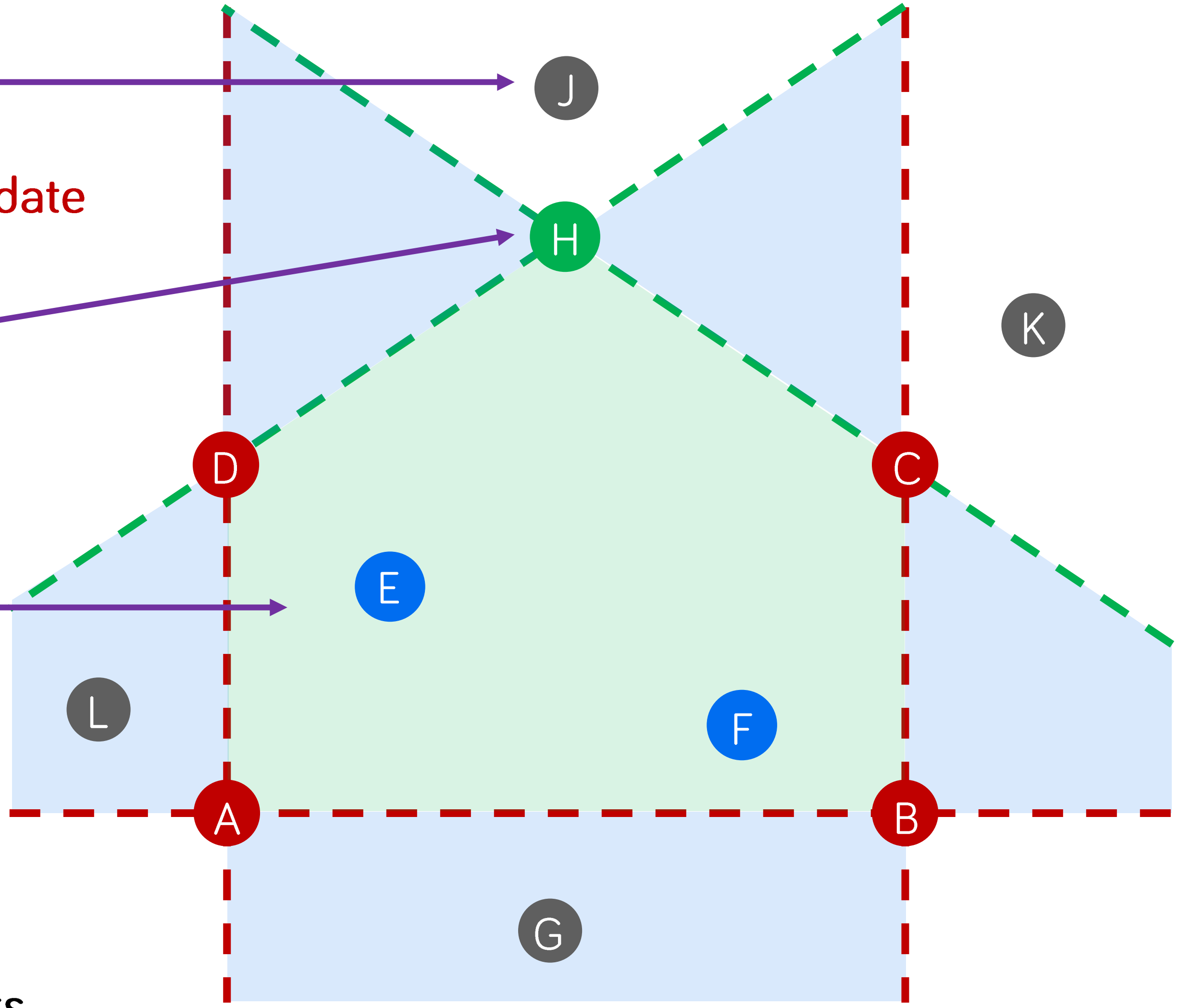
Update set of candidates

($\boldsymbol{J}$) is no longer a candidate

Add extreme point (H)

New pattern

$[A, B, C, \boldsymbol{H}, D]$



Continue enumeration from a candidate point that comes after (H).
$\Longrightarrow$ $not\ G, not\ J,$ $not\ K, \boldsymbol{but\ L}$

Bottom Up

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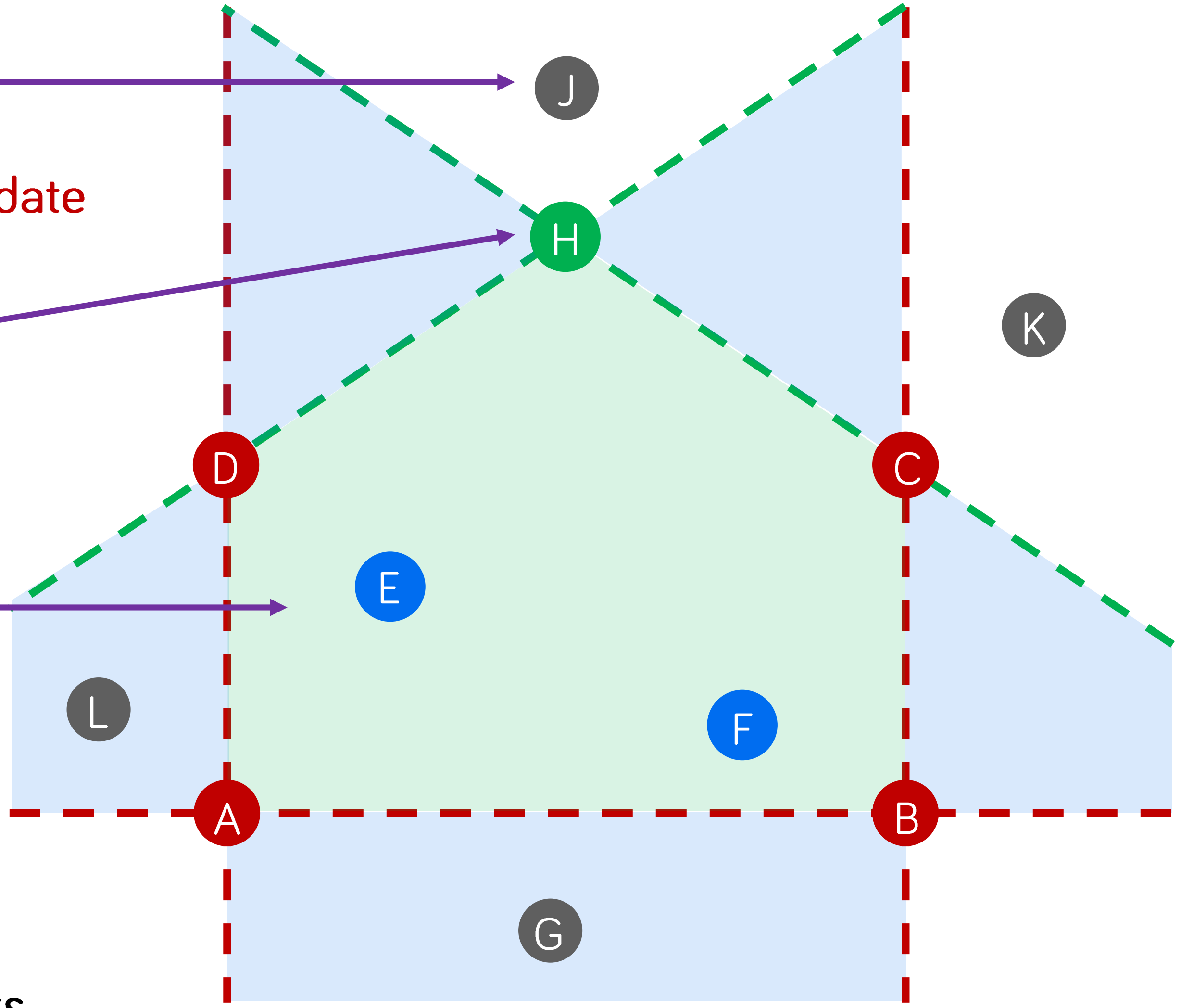


All visited pattern are generated once.
$\Longrightarrow$ no backtracks.

Continue enumeration from a candidate point that comes after (H).
$\Longrightarrow$ $not\ G, not\ J,$
$not\ K,$ $\boldsymbol{but\ L}$

Bottom Up

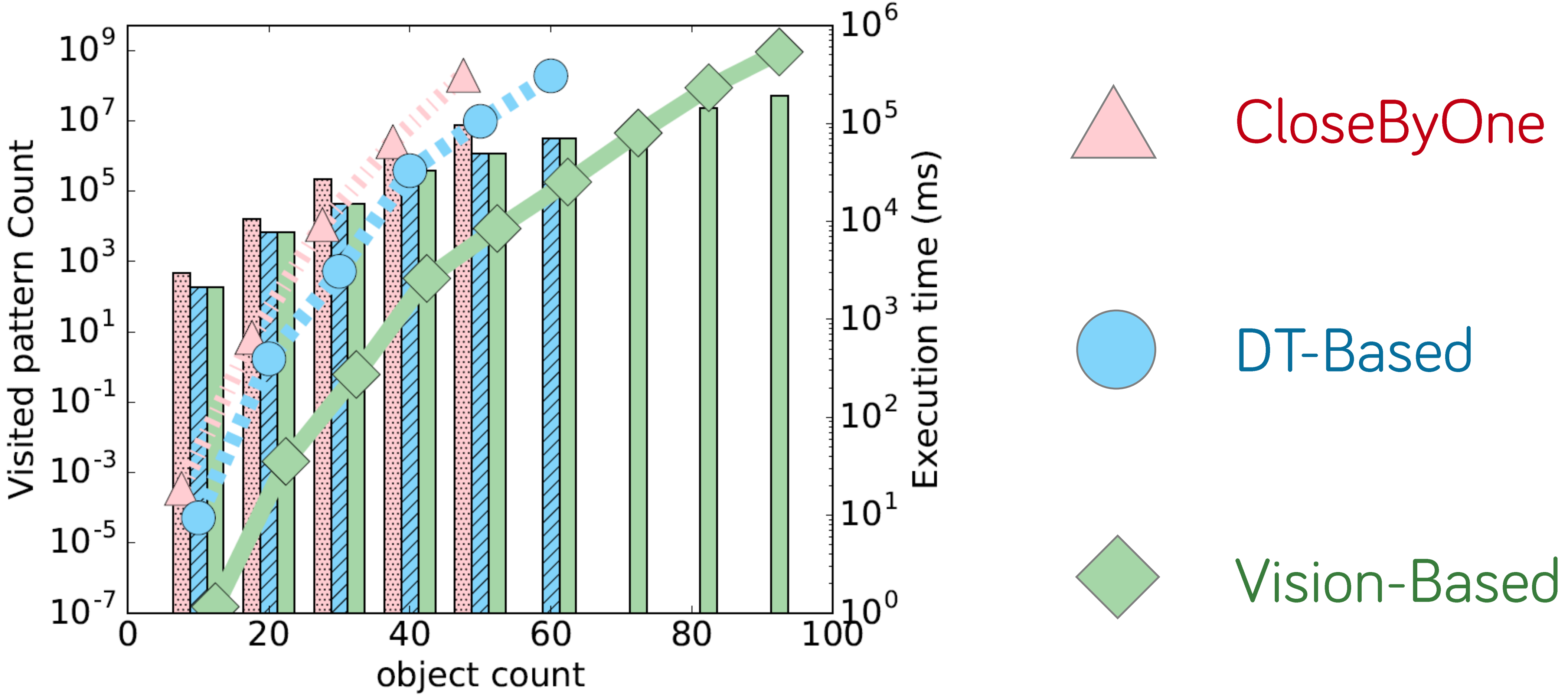
- Points that are in the blue regions are candidate points.

Constraint	CloseByOne (⬆)	DT-Based (⬇)	Vision-Based (⬆)
Min Support	✗	✓	✗
Min Area	✗	✓	✗
Max Area	✓	✗	✓
Min Perimeter	✗	✓	✗
Max Perimeter	✓	✗	✓
Max **complexity***	✗	✗	✓

***Complexity:** number of polygon vertices (or edges).

1. Algorithms
2. Experiments
3. Perspective

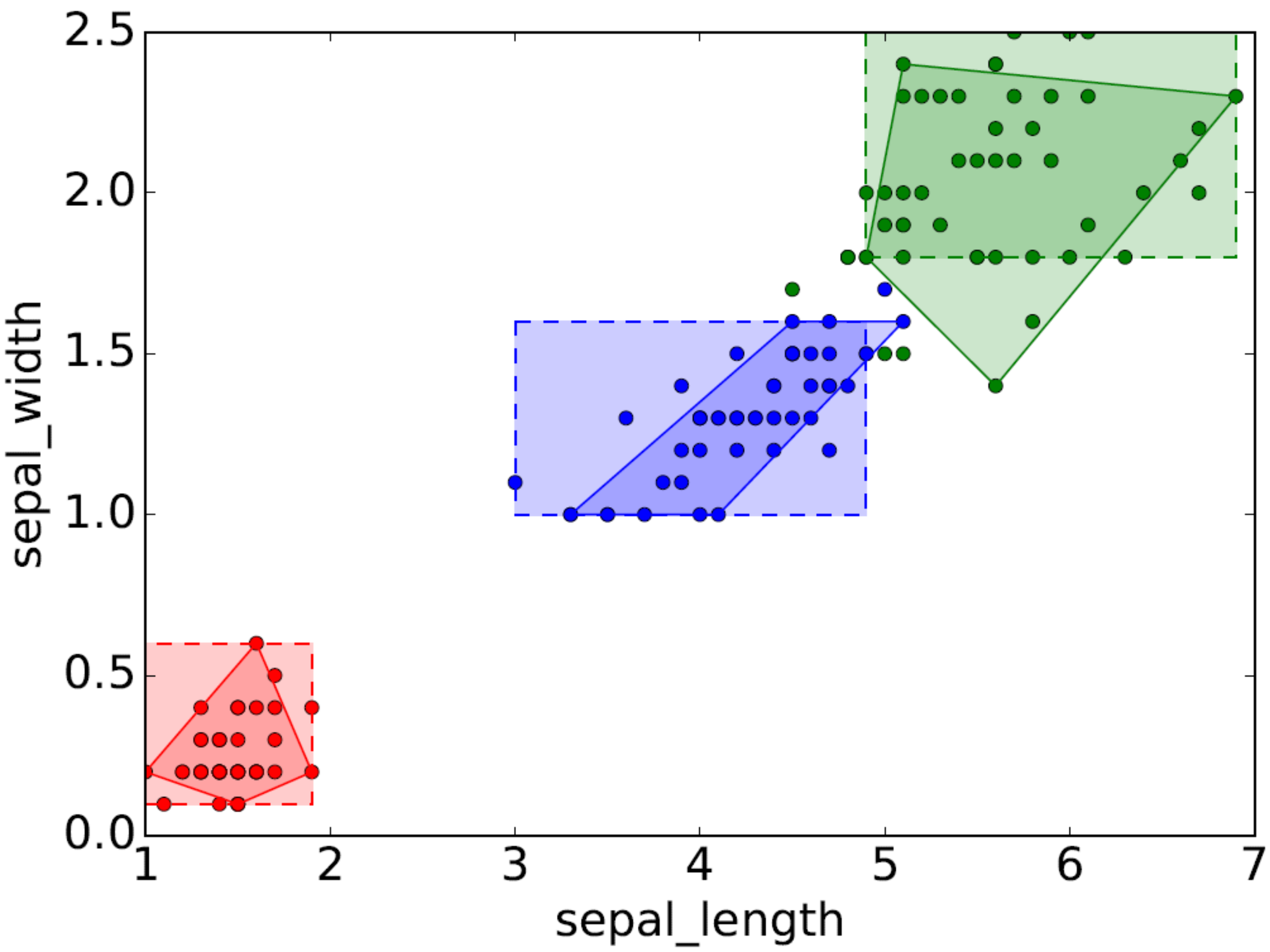
Algorithms Performance



*The used dataset is IRIS dataset projected on sepal width and petal width dimensions.

Interval Pattern VS Convex Polygon Pattern

Top-three homogenous interval patterns and convex polygon patterns with largest support and at most 4 vertices.



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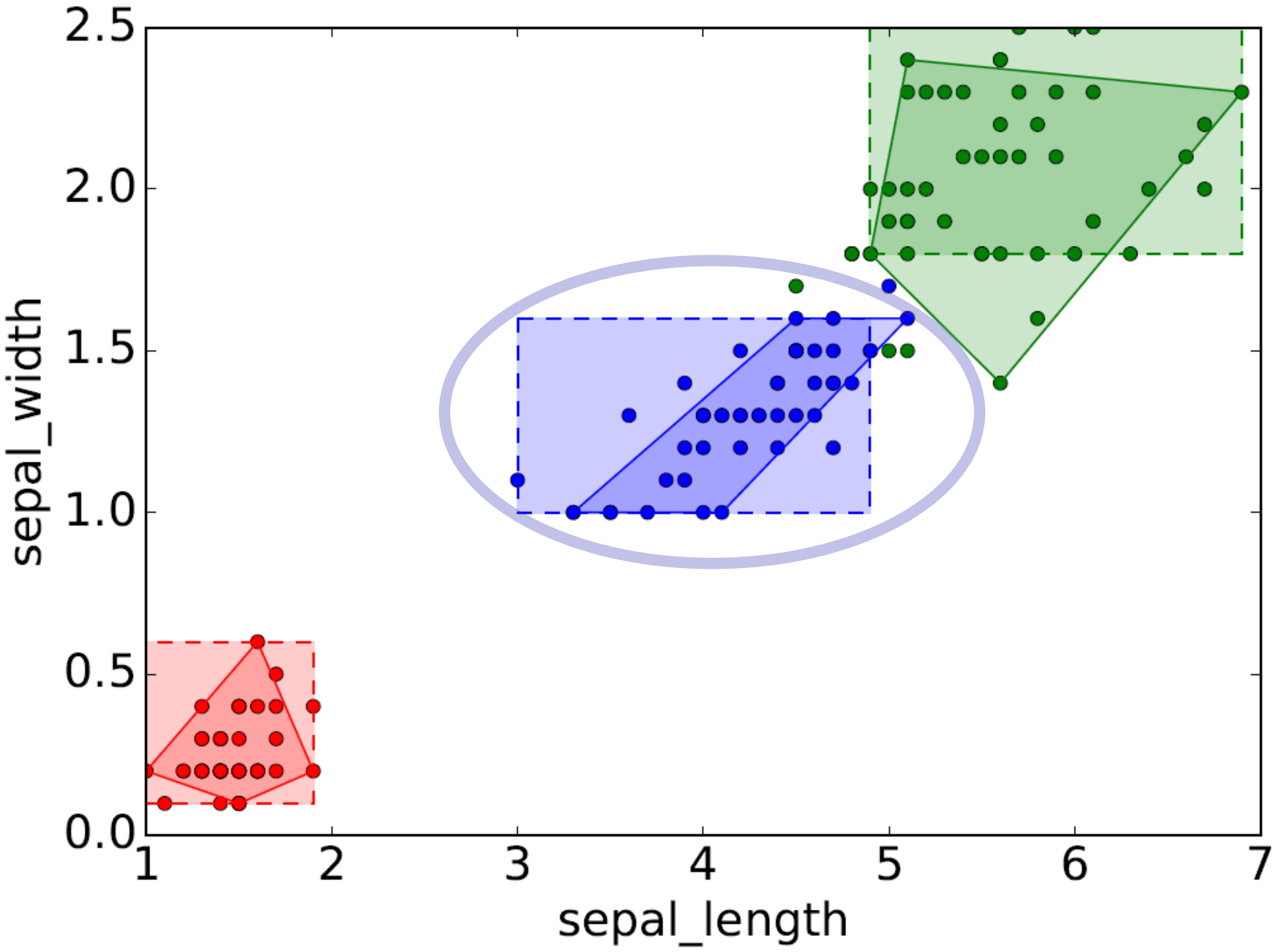
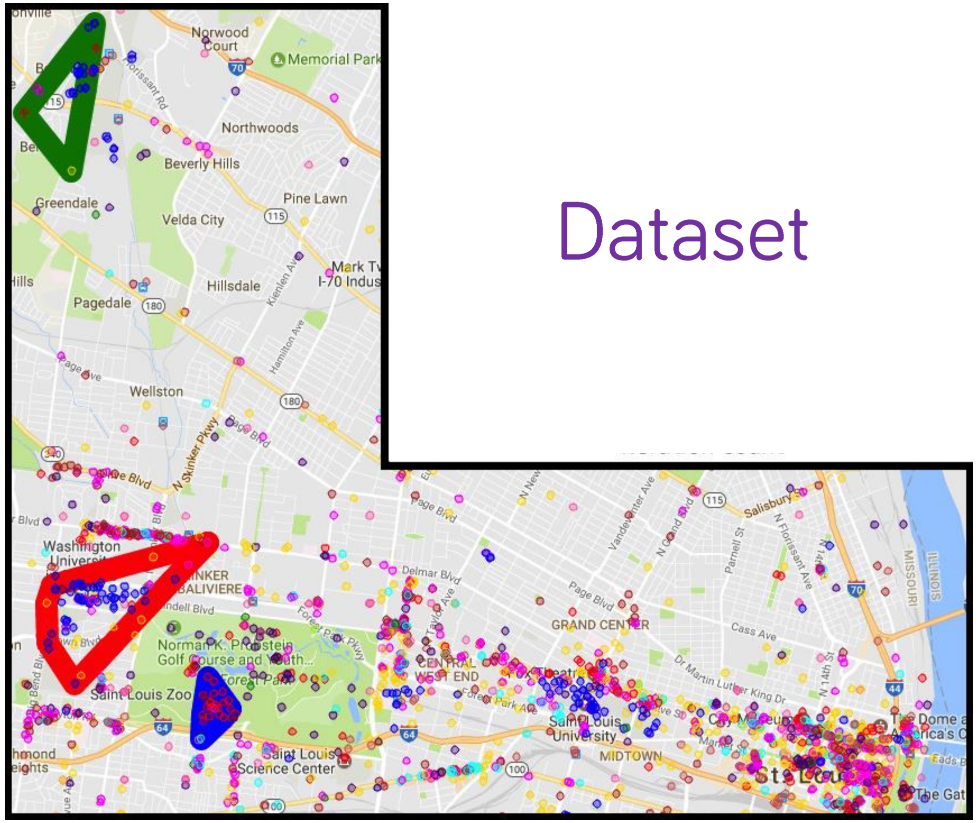


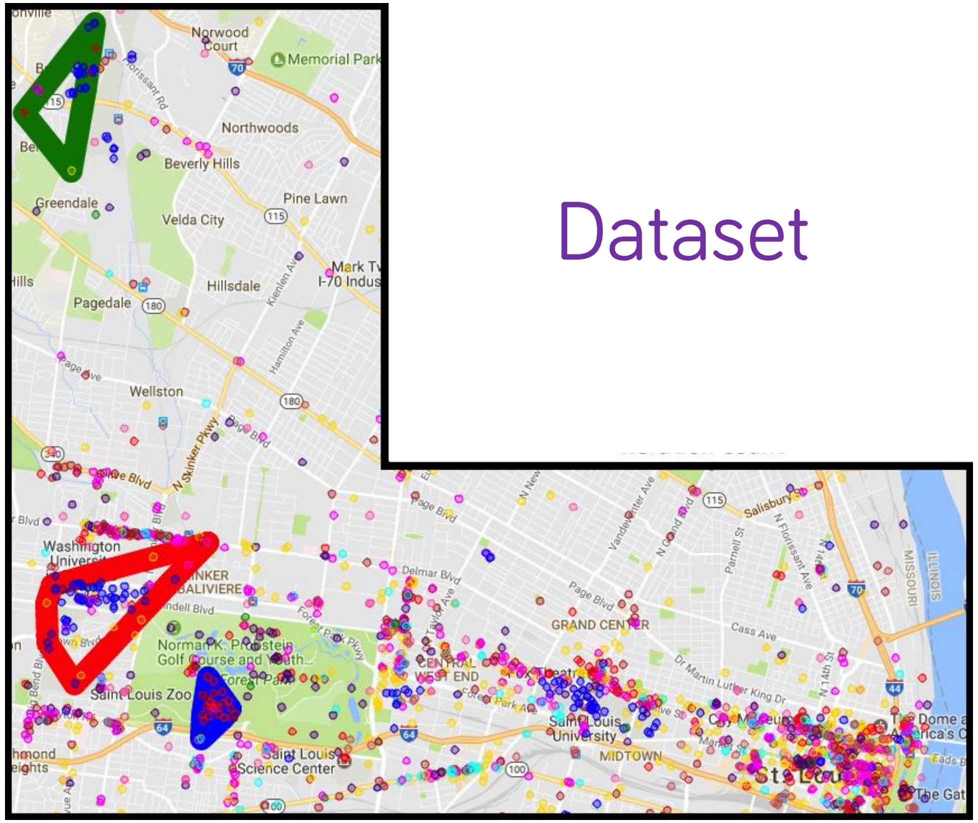
Convex Polygon Pattern in Action



Dataset

Convex Polygon Pattern in Action

Dataset. **St. Louis** town described by **3464** Points of interest (University, shop, ...) collected from *Foursquare*.



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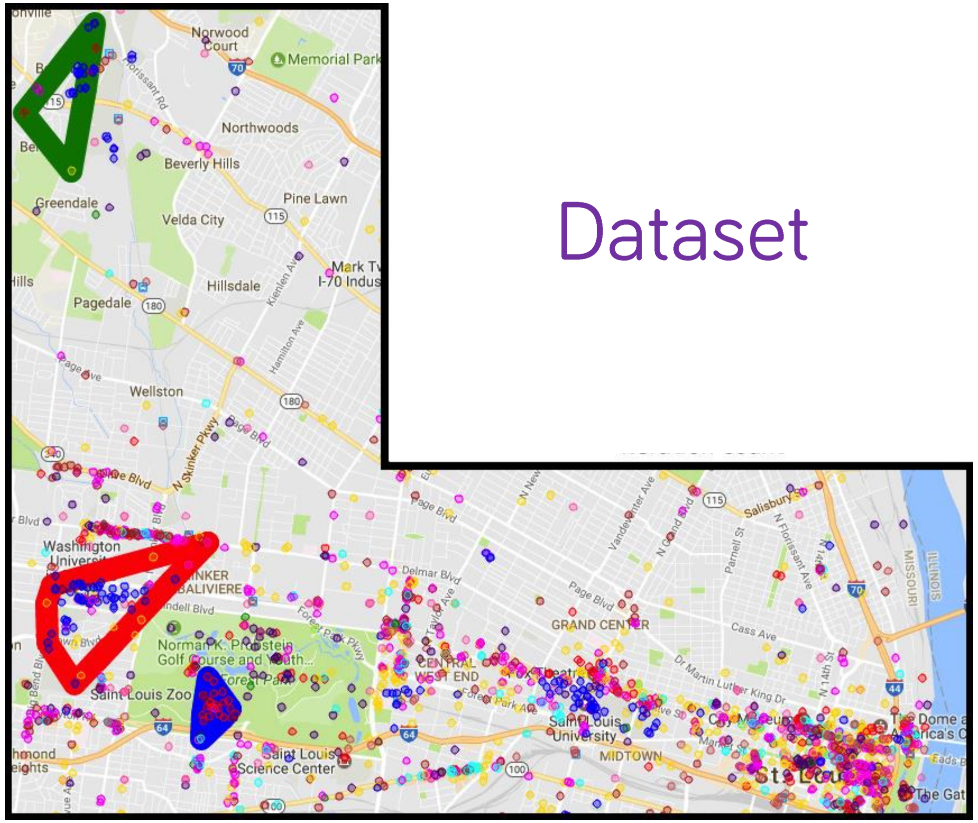

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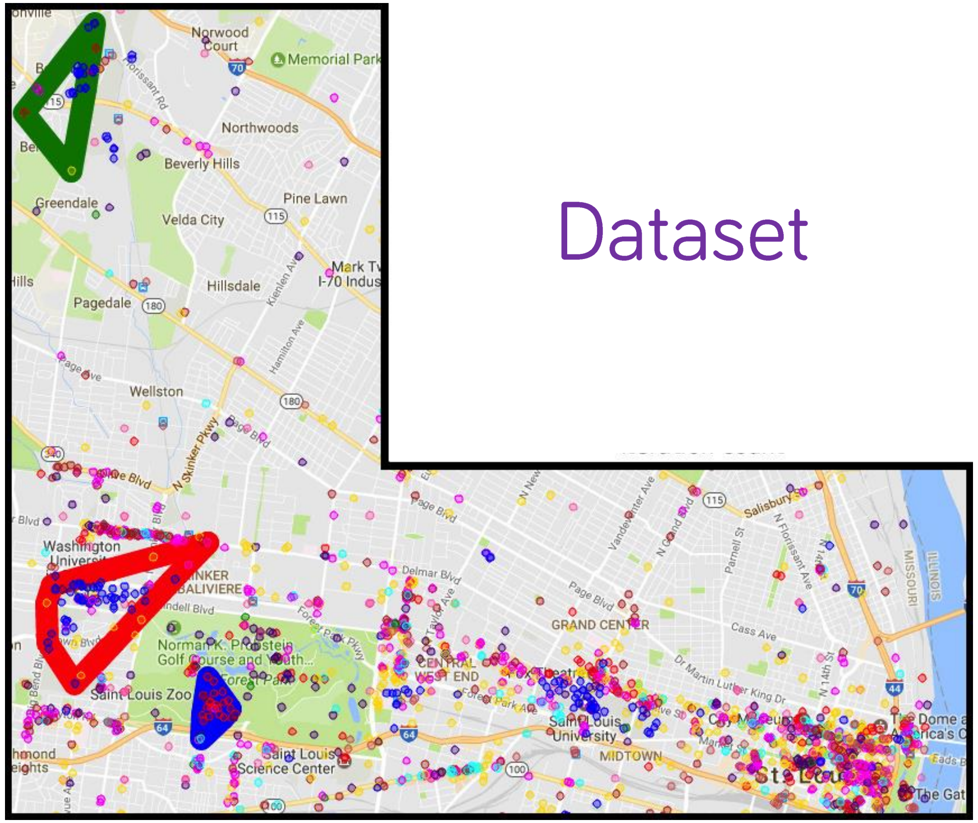
Fact. Running an exhaustive search is almost impossible.



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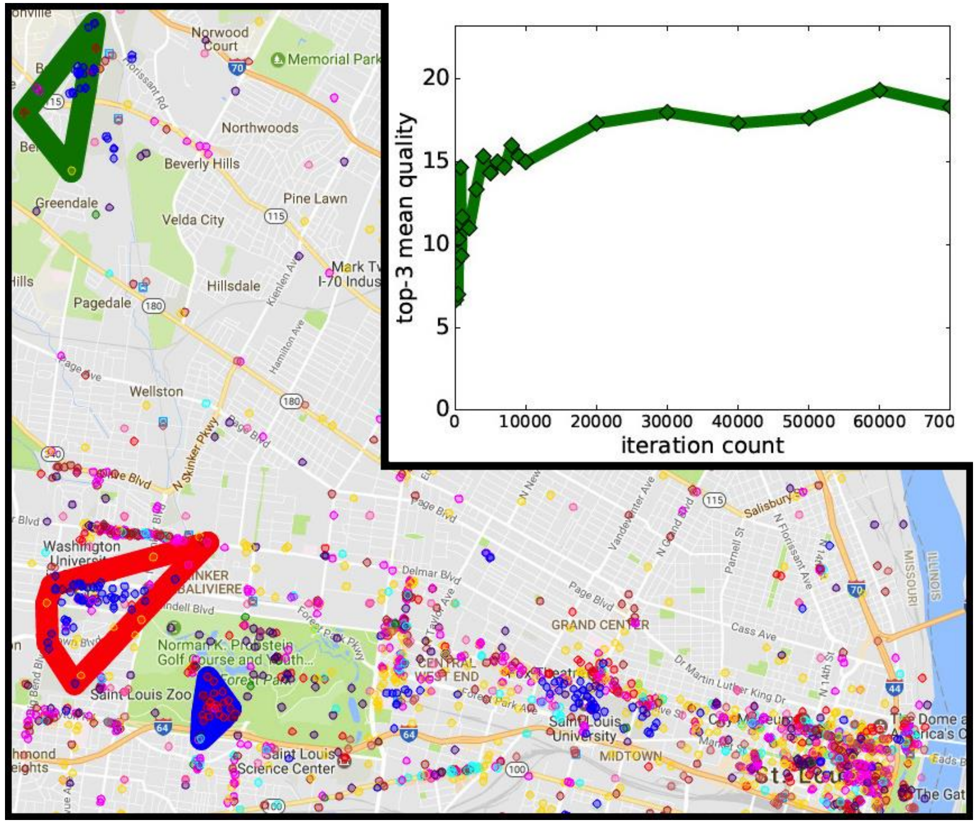
Solution. Make use of heuristics or pattern sampling techniques to find good patterns fast. We made use of the newly proposed Monte-Carlo Tree Search pattern sampling technique [Bosc, G. et al. (**DMKD - minor revision**)] .



Guillaume Bosc, Chedy Raïssi, Jean-François Boulicault, Mehdi Kaytoue
Any-time Diverse Subgroup Discovery with Monte-Carlo Tree Search.
CoRR, abs/1609.08827, 2016.

1. Algorithms
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Perspectives

1. Enumerate these patterns in higher dimensions.

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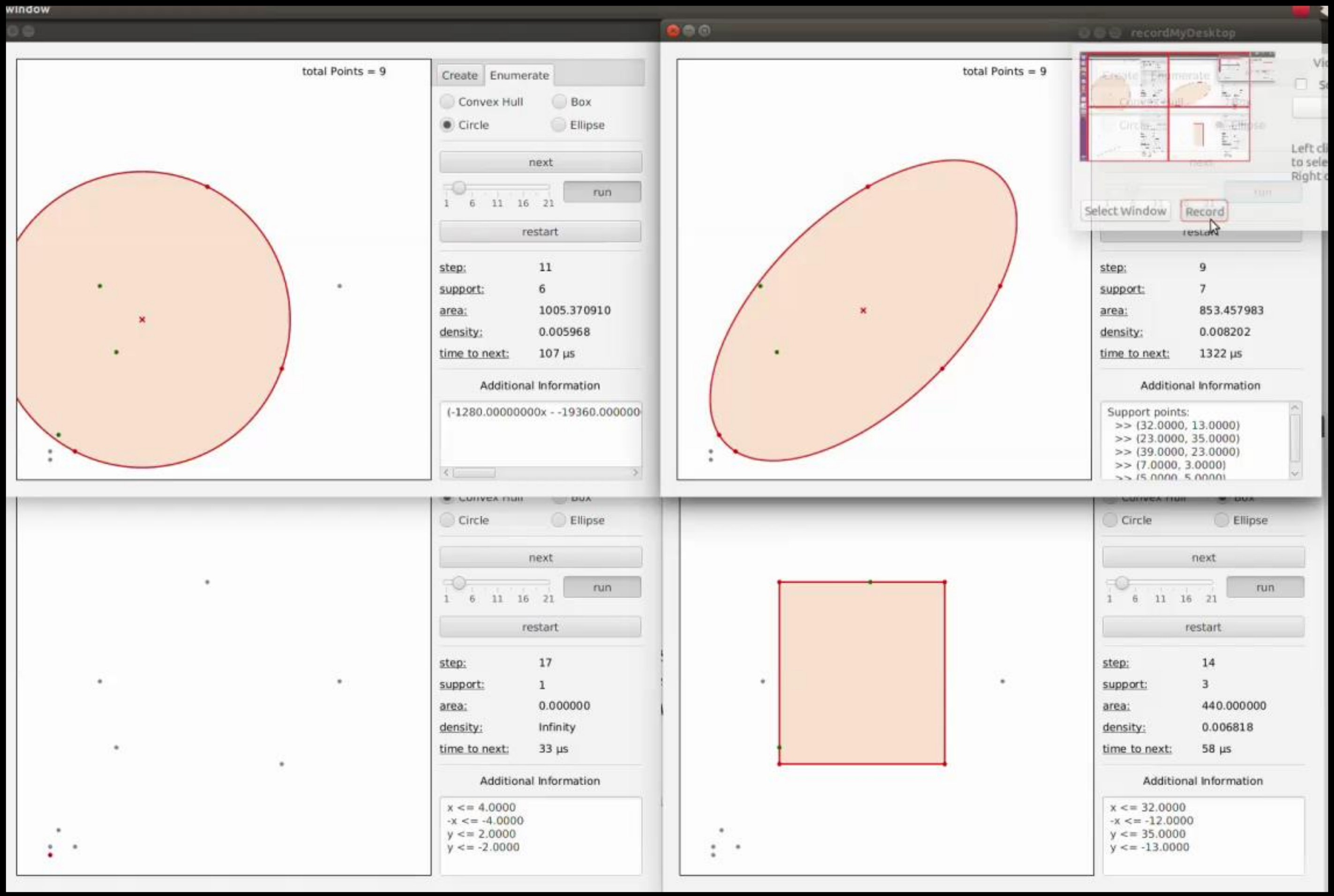
2. Explore and enhance pattern sampling techniques for this new pattern domain.
3. Evaluate predictive performance of this new kind of patterns.
4. Explore other geometric shapes, yet more expressive than interval patterns, but less expensive than convex polygon patterns.

A glimpse on current works ...



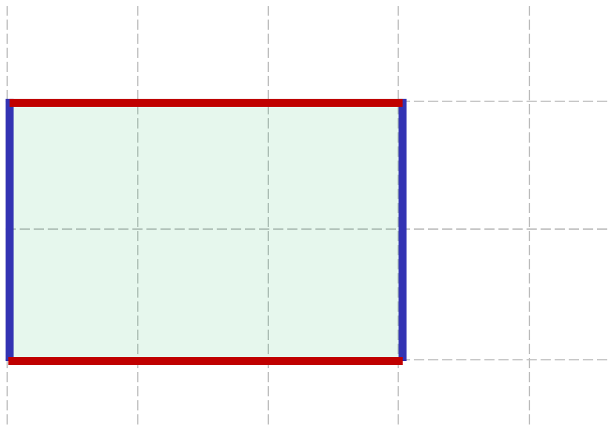
Small discussion (1)

More
Interpretability

More
Expressivity

Small discussion (1)

More Interpretability — More Expressivity



$1 \leq x \leq 4$ ***AND***
$2 \leq y \leq 4$.

Small discussion (1)

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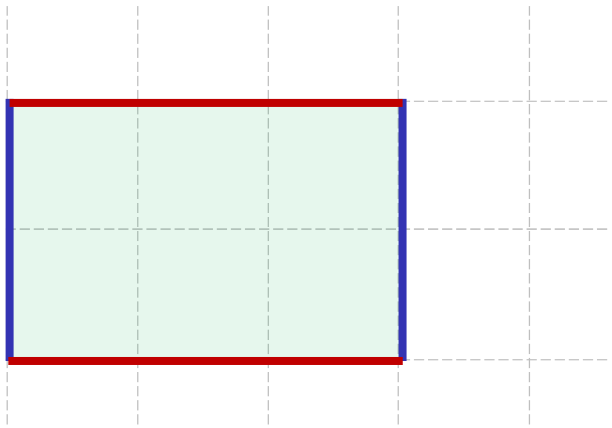

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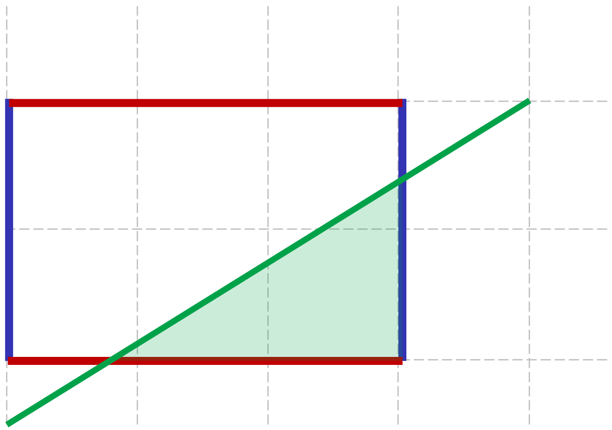

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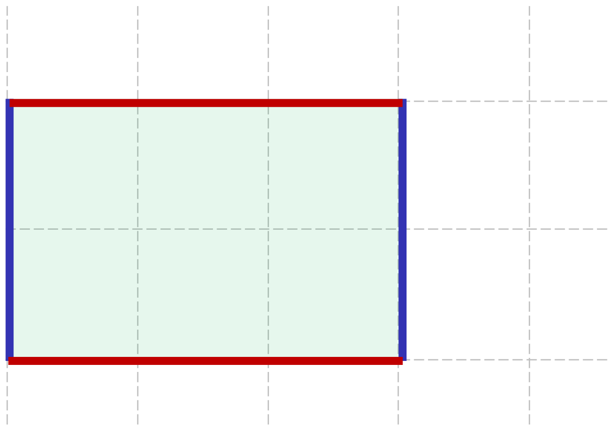

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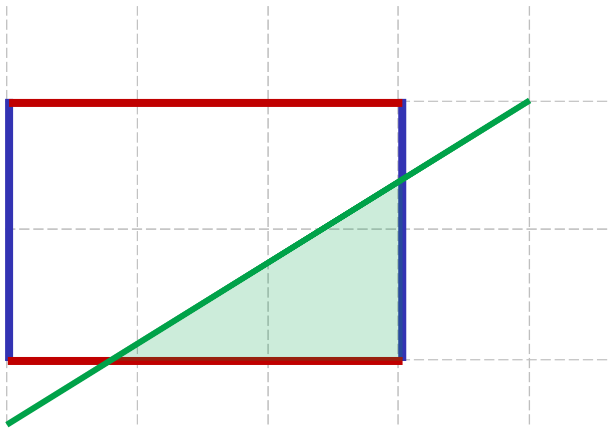

$1 \leq x \leq 4$ $\boldsymbol{AND}$
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$\boldsymbol{x \leq y}$ → Boss salary ≤ Employee salary

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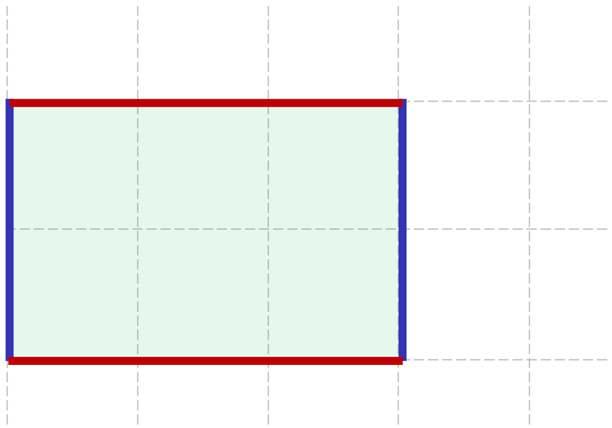

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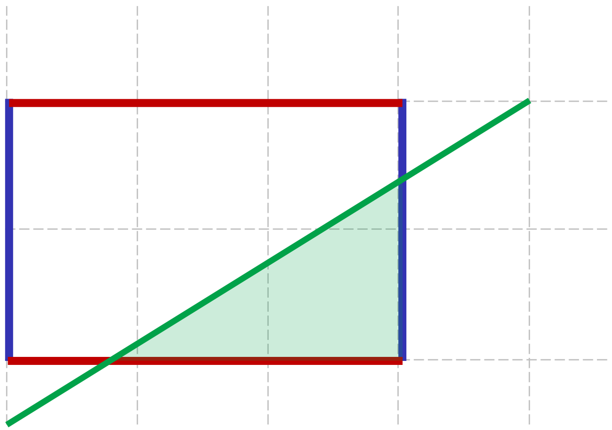

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...

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Small discussion (2)

For finite subsets of $\mathbb{R}^2$ of size n we have **(minimum area)**:

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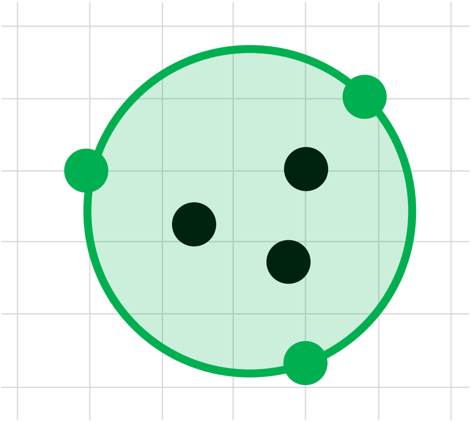

3 point suffice to build a circle

For finite subsets of $\mathbb{R}^2$ of size n we have **(minimum area)**:

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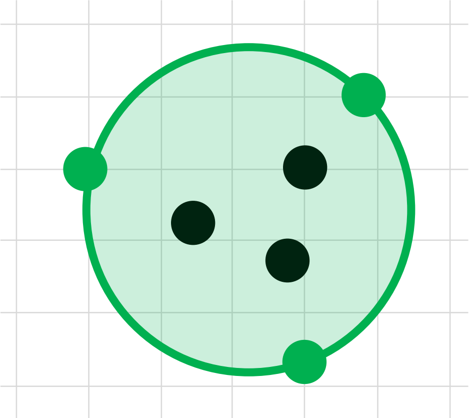

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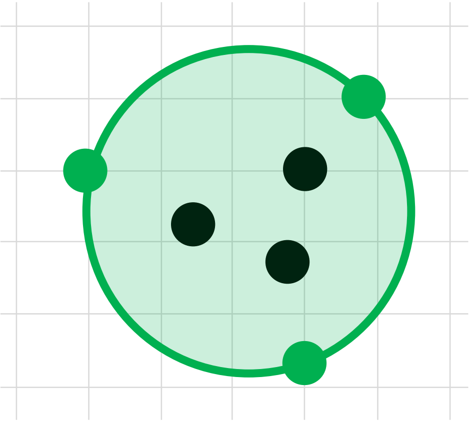

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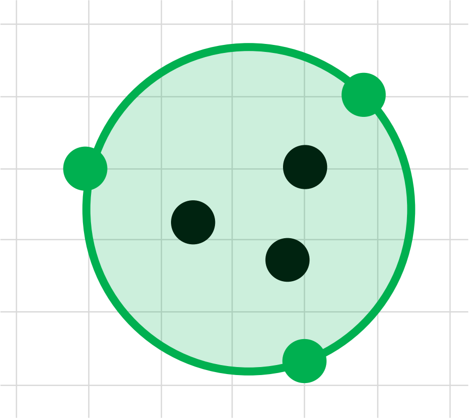

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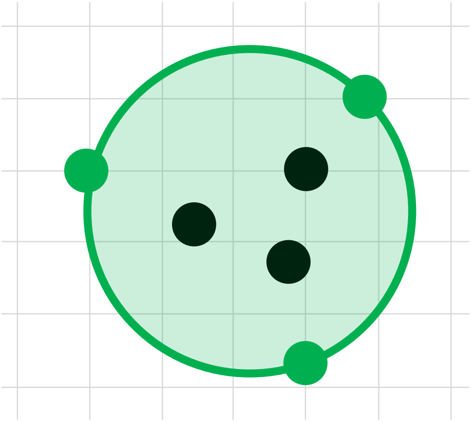

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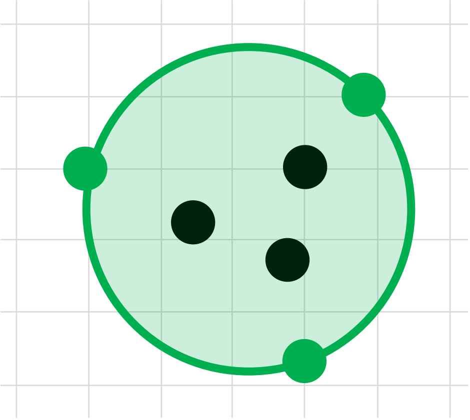

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Not all language are comparable.

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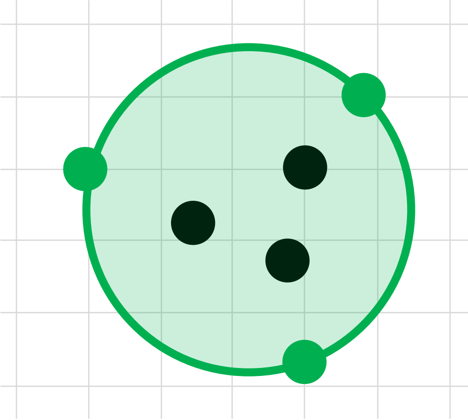

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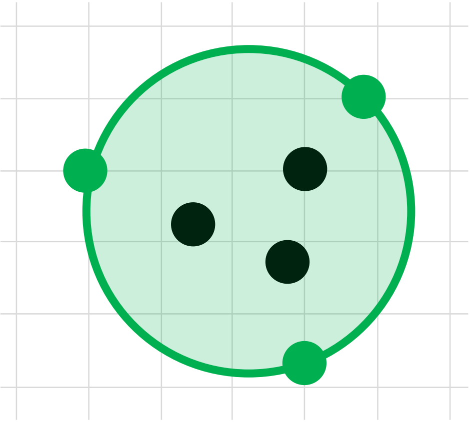

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Not all language are comparable.

- **Convex polygon pattern language** subsume **all convex forms based language.**
- **Rectangle (axis-parallel) pattern language** is not comparable with **ellipsoids pattern language**

Thank you for your attention.
Questions ?

Contact: aimene.Belfodil@insa-lyon.fr
Materials: https://github.com/BelfodilAimene/MiningConvexPolygonPatterns
Paper: https://www.ijcai.org/proceedings/2017/0197.pdf